Please write* *Your name: _____

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. NO calculators are needed.

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(1) There are 52 cards in a standard deck of playing cards. There are 4 *suits*: hearts, spades, diamonds, and clubs (♡♠♢♣). Hearts and diamonds are red while spades and clubs are black. In each suit there are 13 *ranks*: the numbers $2, 3 \ldots, 10$, the three face cards, Jack, Queen, King, and the Ace. Note that Ace is not a face card. Suppose we choose two cards at random, one after another without replacement. Let $A = \{\text{the first card is a King}\}$, $B = \{\text{the second card is a King}\}$. Are these events independent or not? Explain.

Answer: $\mathbb{P}(A) = \mathbb{P}(B) = \dfrac{\binom{4}{1}}{\binom{52}{1}} = \dfrac{4}{52} = \dfrac{1}{13}$, $\mathbb{P}(A \cap B) = \dfrac{\binom{4}{2}}{\binom{52}{2}} = \dfrac{4 \cdot 3}{52 \cdot 51}$. Not independent.

(2) Two dice are simultaneously rolled. Let $A = \{\text{the sum is 7}\}$, $B = \{\text{the first die lands a 5}\}$. Are these events independent or not? Explain.

Answer: $\mathbb{P}(A) = \mathbb{P}(B) = \dfrac{1}{6}$, $\mathbb{P}(A \cap B) = \dfrac{1}{36}$. Independent.

(3) You make successive independent flips of a coin that lands on heads with probability $\frac{1}{2}$. What is the probability that the 4th head appears on the 8th flip?

Answer: $\binom{7}{3} \dfrac{1}{2^8}$

(4) Suppose we choose 2 balls at random, one after another without replacement, from a box containing 3 red and 3 blue balls. Event A is that the first ball is red. Event B is that the second ball is blue. Are events A and B independent? Explain.

Answer: $\mathbb{P}(A) = \mathbb{P}(B) = \dfrac{1}{2}$, $\mathbb{P}(A \cap B) = \dfrac{3^2}{6 \cdot 5}$. Not independent.

(5) On a fictional planet, the year is 300 days long. Every day there maybe rain, or snow, or both, or neither. In a given year, there are 150 days of rain, 150 days of snow, and 100 days of neither. What is the number of days when there is rain and snow?

Answer: $(150 + 150 + 100) - 300 = 100$ days of rain and snow.

(6) On the same planet, on a random day, are rain and snow independent? Explain.

Answer: $\mathbb{P}(\text{rain}) = \mathbb{P}(\text{snow}) = \dfrac{1}{2}$, $\mathbb{P}(\text{rain and snow}) = \dfrac{1}{3}$. Not independent.