Please write* *Your name: _____

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. Calculators are not needed.

. . . . . .

(1) (4 Points) Suppose that X is a random variable with the outcomes $\{0, 1, 2, 3\}$. The corresponding probabilities are given by,

$$\mathbb{P}(X = 0) = \frac{1}{8},\ \mathbb{P}(X = 1) = \frac{3}{8},\ \mathbb{P}(X = 2) = \frac{3}{8},\ \mathbb{P}(X = 3) = \frac{1}{8}$$

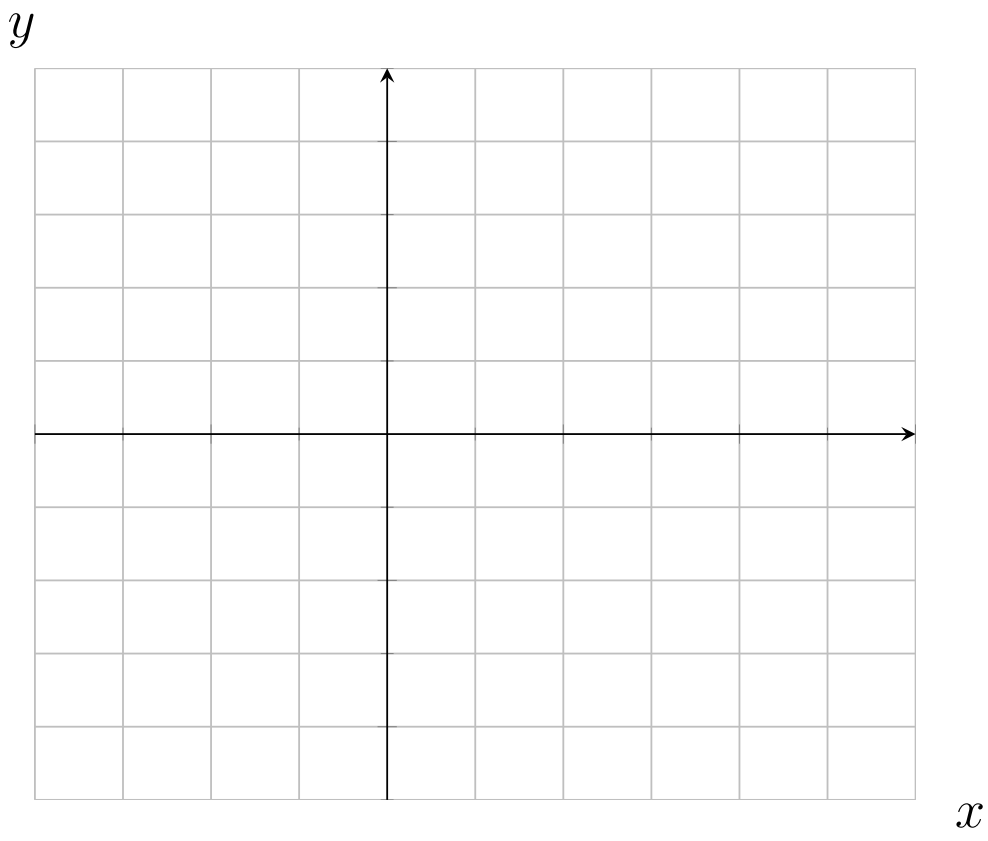
Find its expected value, variance, and standard deviation.

(2) (4 Points) Suppose that X is a random variable with the outcomes $\{0, 1, 2, 3\}$. The corresponding probabilities are given as in question (1) by,

$$\mathbb{P}(X = 0) = \frac{1}{8},\ \mathbb{P}(X = 1) = \frac{3}{8},\ \mathbb{P}(X = 2) = \frac{3}{8},\ \mathbb{P}(X = 3) = \frac{1}{8}$$

Find the cumulative distribution function F_X of X and plot its graph.

$$F_X(x) = \begin{cases} 0, & \text{for } -\infty < x < 0 \\ \_\_\_\_\_, & \text{for } \_\_\_\_\_ \ x \ \_\_\_\_\_ \\ \_\_\_\_\_, & \text{for } \_\_\_\_\_ \ x \ \_\_\_\_\_ \\ \_\_\_\_\_, & \text{for } \_\_\_\_\_ \ x \ \_\_\_\_\_ \\ \_\_\_\_\_, & \text{for } \_\_\_\_\_ \leq x < \infty \end{cases}$$



(3) (1 Points) Toss a fair coin and define

$$X = \begin{cases} 1, & \text{if outcome is heads} \\ 0, & \text{if outcome is tails} \end{cases}$$

Calculate $E[g(X)]$ for $g(X) = 2e^X - 1$.

(4) (1 Points) Does there exist a random variable X such that $E[X] = 4$ and $E[X^2] = 10$? Why or why not? (Hint: Look at its variance)