(1) (4 Points) Suppose that $X$ is a random variable with the outcomes \{0, 1, 2, 3\}. The corresponding probabilities are given by,

$$\Pr(X = 0) = \frac{1}{8}, \Pr(X = 1) = \frac{3}{8}, \Pr(X = 2) = \frac{3}{8}, \Pr(X = 3) = \frac{1}{8}$$

Find its expected value, variance, and standard deviation.
(2) (4 Points) Suppose that $X$ is a random variable with the outcomes \( \{0, 1, 2, 3\} \). The corresponding probabilities are given as in question (1) by,
\[
\begin{align*}
P(X = 0) &= \frac{1}{8}, \\
P(X = 1) &= \frac{3}{8}, \\
P(X = 2) &= \frac{3}{8}, \\
P(X = 3) &= \frac{1}{8}
\end{align*}
\]
Find the cumulative distribution function $F_X$ of $X$ and plot its graph.

\[
F_X(x) = \begin{cases} 
0, & \text{for } -\infty < x < 0 \\
\text{________,} & \text{for } _______ < x < _______ \\
\text{________,} & \text{for } _______ < x < _______ \\
\text{________,} & \text{for } _______ < x < _______ \\
\text{________,} & \text{for } _______ \leq x < \infty
\end{cases}
\]
(3) (1 Points) Toss a fair coin and define

\[ X = \begin{cases} 1, & \text{if outcome is heads} \\ 0, & \text{if outcome is tails} \end{cases} \]

Calculate \( E[g(X)] \) for \( g(X) = 2e^X - 1 \).
(4) (1 Points) Does there exist a random variable X such that $E[X] = 4$ and $E[X^2] = 10$? Why or why not? (Hint: Look at its variance)