In questions on this page, we discuss a rare not inherited genetic mutation which can occur in an individual with probability $p = 10^{-6} = 0.000001$, which is one in a million.

(1) Find the probability that a state of 4 million people have nobody with this particular mutation.

*Answer: $e^{-4}$*

(2) Find the probability that a city of half a million people have at least two people with this particular mutation.

*Answer: $1 - e^{-0.5} - e^{-0.5} \cdot 0.5 = 1 - 1.5e^{-0.5}$*

(3) Find the mean and the standard deviation of the number of mutations in the United States (current population 328 million).

*Answer: $E(X) = 328$, $\text{Var}(X) = \sqrt{328},$*

(4) Patients are coming to a clinic one by one, and each randomly may have a flu with probability 25%, independently one of another. Let $X$ be the number of patients until the first flu case. Find the mean and the standard deviation of $X$. You do not have to simplify your answer.

*Answer: geometric distribution with $p = \frac{1}{4}$, $E(X) = \frac{1}{p} = 4$, $\text{SD}(X) = \sqrt{\frac{1-p}{p^2}} = \sqrt{12}$*

(5) In the same situation, find the probability that $X \geq 3$. You do not have to simplify your answer.

*Answer: $1 - \frac{1}{4} - \frac{3}{16} = \frac{9}{16}$*

(6) Find the mean and the variance of a random variable which is uniformly distributed from 1 to 4.

*Answer: $E(X) = \frac{5}{2}$, $\text{Var}(X) = E((X - E(X))^2) = \frac{1}{4} \cdot 2 \cdot (\frac{1}{4} + \frac{9}{4}) = \frac{5}{4}$,