Please write* *Your name: _____

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. NO calculators are needed.

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In questions on this page, we discuss a rare not inherited genetic mutation which can occur in an individual with probability $p = 10^{-6} = 0.000001$, which is one in a million.

(1) Find the probability that a state of 4 million people have nobody with this particular mutation.

Answer: e^{-4}

(2) Find the probability that a city of half a million people have at least two people with this particular mutation.

Answer: $1 - e^{-.5} - e^{-.5} \cdot 0.5 = 1 - 1.5e^{-.5}$

(3) Find the mean and the standard deviation of the number of mutations in the United States (current population 328 million).

Answer: $\mathbb{E}X = 328$, $\mathrm{SD}(X) = \sqrt{328}$,

(4) Patients are coming to a clinic one by one, and each randomly may have a flu with probability 25%, independently one of another. Let X be the number of patients until the first flu case. Find the mean and the standard deviation of X. You do not have to simplify your answer.

Answer: geometric distribution with $p = \dfrac{1}{4}$, $\mathbb{E}X = \frac{1}{p} = 4$, $\mathrm{SD}(X) = \sqrt{\frac{1-p}{p^2}} = \sqrt{12}$

(5) In the same situation, find the probability that $X \geqslant 3$. You do not have to simplify your answer.

Answer: $1 - \frac{1}{4} - \frac{3}{16} = \dfrac{9}{16}$

(6) Find the mean and the variance of a random variable which is uniformly distributed from 1 to 4.

Answer: $\mathbb{E}X = \dfrac{5}{2}$, $\mathrm{Var}X = \mathbb{E}(X - \mathbb{E}X)^2 = \frac{1}{4} \cdot 2 \cdot (\frac{1}{4} + \frac{9}{4}) = \dfrac{5}{4}$,