***Please write* Your name:** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

In this quiz use the notation $\Phi(x)$ for the distribution function for $\mathcal{N}(0,1)$, that is

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$$

where Z is the standard normal random variable.

(1) Find a formula for $\mathbb{P}(-1 \le X \le 3)$ if X is $\mathcal{N}(-1,4)$. Your answer should include Φ twice. Do not use the normal table in this question.

Answer: $\mathbb{P}(-1 \le X \le 3) = \mathbb{P}(-1 \le -1 + 2Z \le 3) = \mathbb{P}(0 \le Z \le 2) = \Phi(2) - \Phi(0)$

(2) Find the numerical value for $\mathbb{P}(-1 \le X \le 3)$ if X is $\mathcal{N}(-1,4)$. Use the normal table attached in the end of the quiz.

Answer: $\Phi(2) = 0.97725$, $\Phi(0) = 0.5$, $\Phi(2) - \Phi(0) = 0.497725$

(3) Suppose a fair coin is tossed 25 times. Find a formula for a normal approximation for the probability to have at least 15 heads. Your answer should include Φ.

Answer: $\mathbb{P}(X \ge 15) \approx \mathbb{P}(\frac{25}{2} + \frac{5}{2}Z \ge 15) = \mathbb{P}(Z \ge 1) = 1 - \Phi(1)$ ***A more accurate answer:*** $\mathbb{P}(X \ge 15) = \mathbb{P}(X > 14.5) \approx \mathbb{P}(\frac{25}{2} + \frac{5}{2}Z \ge 14.5) = \mathbb{P}(Z \ge 0.8) = 1 - \Phi(0.8)$

(4) Find a numerical approximation for the probability to have at least 15 heads. Use the normal table attached in the end of the quiz.

Answer: $1 - 0.84134 = 0.15866$ ***A more accurate answer:*** $1 - 0.78814 = 0.21186$

(5) Let S_n be the number of heads in n coin tosses. How many times do you need to toss a coin so that standard deviation of S_n is 5?

Answer: $n = 100$

(6) For this value of n, estimate the probability to have at least 40 heads using the table.

Answer: $\mathbb{P}(X \ge 40) \approx \mathbb{P}(50 + 5Z \ge 40) = \mathbb{P}(Z \ge -2) = \Phi(2) \approx 0.97725$
A more accurate answer: $\mathbb{P}(X \ge 40) = \mathbb{P}(X > 39.5) \approx \mathbb{P}(50 + 5Z > 39.5) = \mathbb{P}(Z > -2.1) = \Phi(2.1) \approx 0.98214$