In this quiz use the notation $\Phi(x)$ for the distribution function for $\mathcal{N}(0, 1)$, that is

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$$

where $Z$ is the standard normal random variable.

(1) Find a formula for $\mathbb{P}(-1 \leq X \leq 3)$ if $X$ is $\mathcal{N}(-1, 4)$. Your answer should include $\Phi$ twice. Do not use the normal table in this question.

(2) Find the numerical value for $\mathbb{P}(-1 \leq X \leq 3)$ if $X$ is $\mathcal{N}(-1, 4)$. Use the normal table attached in the end of the quiz.
(3) Suppose a fair coin is tossed 25 times. Find a formula for a normal approximation for the probability to have at least 15 heads. Your answer should include Φ.

(4) Find a numerical approximation for the probability to have at least 15 heads. Use the normal table attached in the end of the quiz.
(5) Let $S_n$ be the number of heads in $n$ coin tosses. How many times do you need to toss a coin so that standard deviation of $S_n$ is 5?

(6) For this value of $n$, estimate the probability to have at least 40 heads using the table.

(End of the quiz, the normal table is on the next page)