Please write **Your name:** _____

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. Calculators are not needed.

. . . .

(1) (4 Points) Suppose that X is a random variable with the outcomes $\{0, 1, 2, 3\}$. The corresponding probabilities are given by,

$$\mathbb{P}(X = 0) = \frac{1}{8}, \ \mathbb{P}(X = 1) = \frac{3}{8}, \ \mathbb{P}(X = 2) = \frac{3}{8}, \ \mathbb{P}(X = 3) = \frac{1}{8}$$

Find its expected value, variance, and standard deviation.

$$E(X) = \frac{12}{8} = \frac{3}{2}$$

$$E(X^2) = \frac{24}{8} = 3$$

$$Var(X) = E(X^2) - (EX)^2$$

$$= 3 - \frac{9}{4} = \frac{3}{4}$$
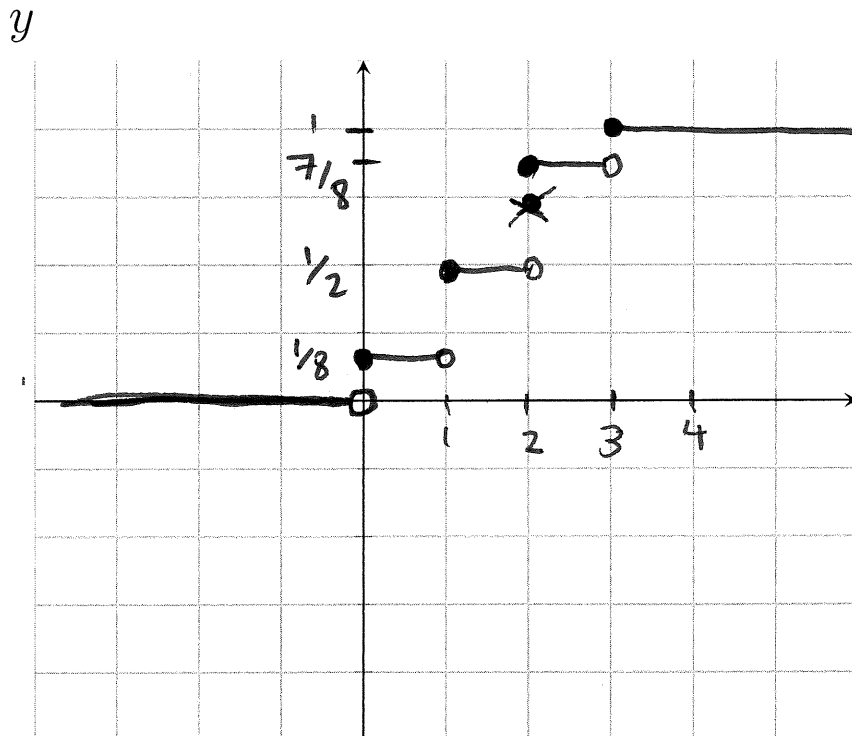
$$SD(X) = \frac{\sqrt{3}}{2}$$

(2) (4 Points) Suppose that X is a random variable with the outcomes $\{0, 1, 2, 3\}$. The corresponding probabilities are given as in question (1) by,

$$\mathbb{P}(X = 0) = \frac{1}{8}, \ \mathbb{P}(X = 1) = \frac{3}{8}, \ \mathbb{P}(X = 2) = \frac{3}{8}, \ \mathbb{P}(X = 3) = \frac{1}{8}$$

Find the cumulative distribution function F_X of X and plot its graph.

$$F_X(x) = \begin{cases} 0, & \text{for } -\infty < x < 0 \\ 1/8, & \text{for } 0 \le x < 1 \\ 1/2, & \text{for } 1 \le x < 2 \\ 7/8, & \text{for } 2 \le x < 3 \\ 1, & \text{for } 3 \le x < \infty \end{cases}$$

(3) (1 Points) Toss a fair coin and define

$$X = \begin{cases} 1, & \text{if outcome is heads} \\ 0, & \text{if outcome is tails} \end{cases}$$

Calculate $E[g(X)]$ for $g(X) = 2e^X - 1$.

$$E(2e^X - 1) = 2E(e^X) - 1$$

$$E(e^X) = \frac{1}{2}e^0 + \frac{1}{2}e^1 = \frac{1}{2}(1 + e)$$

$$\Rightarrow \quad E(2e^X - 1) = 2 \cdot \frac{1}{2}(1 + e) - 1$$

$$= 1 + e - 1 = e$$

(4) (1 Points) Does there exist a random variable X such that $E[X] = 4$ and $E[X^2] = 10$? Why or why not?
(Hint: Look at its variance)

$$Var(X) = E(X^2) - E(X)^2$$
$$= 10 - 16 = -6$$

Does Not exist as $Var(X) \geq 0$