***Please write* Your name:** _____

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. NO calculators are needed.

(1) A secret code must contain, in any order, exactly 1 letter and exactly 3 digits. How many different four-symbol codes are possible?

$4 \cdot 26 \cdot 10^3$

(2) Answer the same question if all the digits must be different.

$4 \cdot 26 \cdot 10 \cdot 9 \cdot 8$

(More on the back)

(3) What is the probability that if we roll 2 dice, the sum is 3?

$$\frac{1}{18}$$

(4) What is the probability that if we roll 3 dice, the sum is 4?

$$\frac{3}{6^3}$$

(More on the back)

(5) Two dice are simultaneously rolled. Let $A = \{\text{the sum is } 3\}$, $B = \{\text{the first die lands a } 2\}$. Are these events independent or not? Explain.

$P(A) = \frac{1}{18}$ $\quad$ $P(B) = \frac{1}{6}$

$\frac{1}{36} = P(A \cap B) \neq P(A) \cdot P(B) = \frac{1}{18} \cdot \frac{1}{6}$

(6) You make successive independent flips of a coin that lands on heads with probability $\frac{1}{2}$. What is the probability that the 2nd head appears on the 4th flip?

$3 \cdot \frac{1}{2^4}$

(More on the back)

(7) Suppose we choose 2 balls at random, one after another without replacement, from a box containing 5 red and 3 blue balls. What is the probability both balls are red?

$$\frac{\binom{5}{2}}{\binom{8}{2}} = \frac{5}{14}$$

(8) Suppose we choose 2 balls at random, one after another without replacement, from a box containing 5 red and 3 blue balls. What is the conditional probability that second ball is red, given that the first one is red?

$$\frac{4}{7}$$

(More on the back)

(9) Suppose that X is a random variable with the outcomes $\{0, 1, 2, 3\}$ and the corresponding probabilities given by

$$\mathbb{P}(X = 0) = \frac{1}{6}, \quad \mathbb{P}(X = 1) = 0, \quad \mathbb{P}(X = 2) = \frac{1}{2}, \quad \mathbb{P}(X = 3) = \frac{1}{3}$$

(a) Find the expected value $\mathbb{E}(X)$

$EX = 2$

(b) Find $\mathbb{E}(X^2)$

$EX^2 = 5$

(c) Find the variance $\mathrm{Var}(X)$.

$Var(X) = 1$

(d) Find the formula for the standard deviation of X.

$SD(X) = 1$

(End of the test)