

(1) (a) Let a fair dice be thrown X times until the first time 6 appears. Find $\mathbb{E}X$.

Answer: 6

(b) Find $\text{Var}X$. Do not simplify your answer.

Answer: $\frac{5}{36} = 30$

(c) Find $\mathbb{P}(X \geq 3)$. Do not simplify your answer.

Answer: $1 - \frac{1}{6} - \frac{5}{36}$

(2) Let X be the number of tails in 400 fair coin tosses. Find a normal approximation for $\mathbb{P}(X \geq 220)$. Use the table to find the approximate numerical value for this probability.

Answer: $1 - \Phi(2) \approx 1 - 0.97725 = 0.02275$

With the continuity correction the more precise answer is $1 - \Phi(1.95) \approx 1 - 0.97441 = 0.02559$

(3) Suppose X has the following p.d.f.

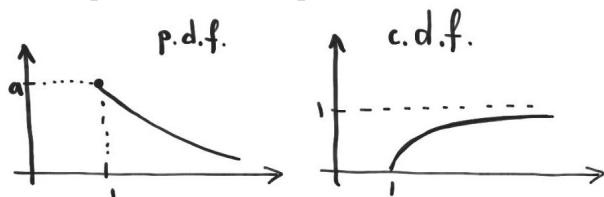
$$f(x) = \begin{cases} \frac{a}{x^3} & \text{if } 1 \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Find a . **Answer:** $a = 2$

(b) Find $\mathbb{E}X$. **Answer:** $\mathbb{E}X = 2$

(c) Find $\text{Var}X$. **Answer:** DNE

(d) Make pictures of the p.d.f. and c.d.f., and label which is which. **Answer:**



(4) Consider random variables X and Y with the joint probability density function

$$f(x, y) = \begin{cases} \frac{ay}{x^2} & \text{if } 1 \leq x \leq 2, 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find a **Answer:** $a = 4$

(b) Find the marginal p.d.f. $f_X(x)$ **Answer:** $f_X(x) = \frac{2}{x^2}$ if $1 \leq x \leq 2$, and 0 otherwise

(c) Find the marginal p.d.f. $f_Y(y)$. **Answer:** $f_Y(y) = 2y$ if $0 \leq y \leq 1$, and 0 otherwise

(d) Are X and Y independent? Explain.

Answer: yes, because $\frac{4y}{x^2} = \frac{2}{x^2} \cdot 2y$ and so $f(x, y) = f_X(x) \cdot f_Y(y)$