Please write **Your name:**

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. NO calculators are needed.

In this quiz use the notation $\Phi(x)$ for the distribution function for $\mathcal{N}(0, 1)$, that is

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = P(Z < x)$$

where $Z$ is the standard normal random variable.

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<th>points for this quiz</th>
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<td>current grade based on the standard curve with the lowest quiz dropped</td>
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<td>preliminary projection of the final grade (not official)</td>
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(1) (a) Let a fair dice be thrown $X$ times until the first time 6 appears. Find $E(X)$.

(b) Find $\text{Var}(X)$. Do not simplify your answer.

(c) Find $P(X \geq 3)$. Do not simplify your answer.
(2) Let $X$ be the number of tails in 400 fair coin tosses. Find a normal approximation for $\mathbb{P}(X \geq 220)$. Use the table to find the approximate numerical value for this probability.

Write your answer here:
(3) Suppose $X$ has the following p.d.f.

$$f(x) = \begin{cases} \frac{a}{x^3} & \text{if } 1 \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $a$.

(b) Find $\mathbb{E}X$. 

(more on the next page)
(c) Find VarX.

(d) Make pictures of the p.d.f. and c.d.f., and label which is which.
(4) Consider random variables $X$ and $Y$ with the joint probability density function

$$f(x, y) = \begin{cases} \frac{ay}{x^2} & \text{if } 1 \leq x \leq 2, \ 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $a$

(b) Find the marginal p.d.f. $f_X(x)$

(more on the next page)
(c) Find the marginal p.d.f. $f_Y(y)$.

(d) Are $X$ and $Y$ independent? Explain.

(End of the test, the normal table and the table of distributions are on the next pages)