***Please write* Your name:** Sasha Teplyaev

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

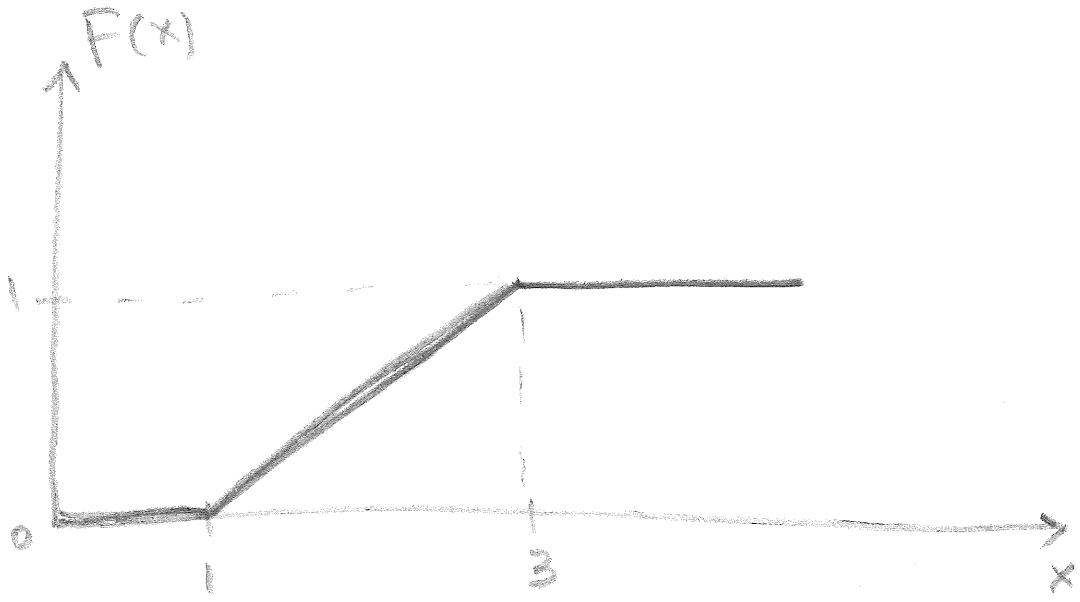
Let X be a uniform random variable on the interval $[1, 3]$.

(1) Find the formula, with cases, for the c.d.f. $F_X(x)$.

$$f(x) = \begin{cases} \frac{1}{2} & \text{if } 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} 0 & \text{if } x \le 1 \\ \frac{1}{2}x - \frac{1}{2} & \text{if } 1 \le x \le 3 \\ 1 & \text{if } x \ge 3 \end{cases}$$

(2) Sketch the plot for the c.d.f. $F_X(x)$.



(more questions on the next page)

Let again X be a uniform random variable on the interval $[1, 3]$.

(3) Find $\mathbb{E}e^X$.

$$\mathbb{E}e^X = \int e^x f(x)\,dx = \int_1^3 \frac{1}{2} e^x\,dx$$

$$= \frac{1}{2} e^x \Big|_1^3 = \frac{1}{2}(e^3 - e)$$

(4) Find $\mathbb{E}\frac{1}{X}$.

$$\mathbb{E}\frac{1}{X} = \int \frac{1}{x} f(x)\,dx = \int_1^3 \frac{1}{2x}\,dx$$

$$= \frac{1}{2} \log(x) \Big|_1^3 = \frac{1}{2} \log 3$$

(more questions on the next page)

Let again X be a uniform random variable on the interval $[1, 3]$ and $Y = \frac{1}{X}$.

(5) Find the c.d.f. $F_Y(y)$ of Y. *Hint: it may be useful if first you find the range of Y, and use cases to define the c.d.f.*

$1 \le X \le 3 \Rightarrow \frac{1}{3} \le Y \le 1$

$F_Y(y) = P(Y \le y) = P\left(\frac{1}{X} \le y\right) =$

$= P\left(X \ge \frac{1}{y}\right) = 1 - F_X\left(\frac{1}{y}\right) = \begin{cases} 0 & \text{if } y \le \frac{1}{3} \\ \frac{3}{2} - \frac{1}{2y} & \text{if } \frac{1}{3} < y \le 1 \\ 1 & \text{if } y \ge 1 \end{cases}$

(6) Find the p.d.f. $f_Y(y)$ of Y.

$f_Y(y) = F_Y'(y) = \begin{cases} \frac{1}{2y^2} & \text{if } \frac{1}{3} < y \le 1 \\ 0 & \text{otherwise} \end{cases}$

(End of the quiz)