MATH 3160 - Probability - Fall 2018 **Quiz 10, Friday November 2**

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Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

Let $X = \text{Exp}(2)$ and $Y = \text{Exp}(3)$ be two independent exponential random variables with parameters 2 and 3.

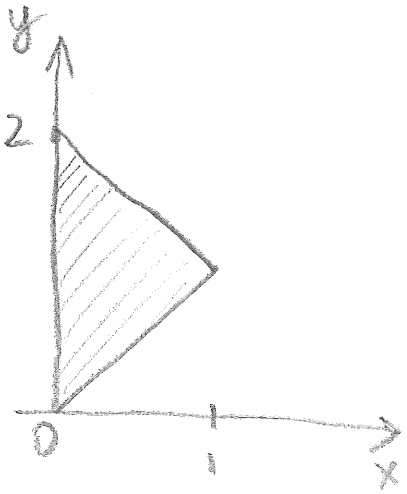
(1) Find the joint probability density function $f(x, y)$.

$$f_X(x) = 2e^{-2x}$$

$$f_Y(y) = 3e^{-3y}$$

$$f(x,y) = 6e^{-2x-3y}$$

(2) In the same setting, write the double integral for $\mathbb{P}(X < Y < 2 - X)$. You do not have to evaluate the integral, but set up the limits of integration carefully.



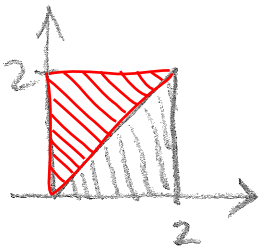
answer:

$$\int_0^1 \int_x^{2-x} 6e^{-2x-3y}\, dy\, dx$$

(more questions on the next page)

Consider X and Y given by the joint density

$$f(x,y) = \begin{cases} \frac{1}{2} & \text{if } 0 \le x \le y \le 2 \\ 0 & \text{otherwise.} \end{cases}$$



(3) Find the marginal p.d.f. $f_X(x)$.

$$f_X(x) = \int f(x,y)\,dy = \int_x^2 \frac{1}{2}\,dy = \frac{y}{2}\Big|_x^2$$

$$f_X(x) = \frac{2-x}{2} = \begin{cases} 1 - \frac{x}{2} & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(4) Find the marginal p.d.f. $f_Y(y)$.

$$f_Y(y) = \int f(x,y)\,dx = \int_0^y \frac{1}{2}\,dx = \frac{y}{2}$$

$$f_Y(y) = \begin{cases} \frac{y}{2} & \text{if } 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(more questions on the next page)

Suppose the joint density function of the random variable X_1 and X_2 is

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1, 0 < x_2 < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(5) Let $Y_1 = \dfrac{1}{X_1}$ and $Y_2 = X_1 X_2$. What is the joint density function of Y_1 and Y_2?

$$J = \det \begin{vmatrix} -\frac{1}{x_1^2} & 0 \\ x_2 & x_1 \end{vmatrix} = -\frac{1}{x_1} = -y_1$$

$$x_1 = \frac{1}{y_1} \qquad x_2 = \frac{y_2}{x_1} = y_1 y_2$$

$$f_{Y_1,Y_2}(y_1, y_2) = \frac{x_1 + x_2}{\frac{1}{x_1}} = \frac{\frac{1}{y_1} + y_1 y_2}{y_1} = \frac{1}{y_1^2} + y_2$$

$$f_{Y_1,Y_2} = \begin{cases} \frac{1}{y_1^2} + y_2 & \text{if } y_1 > 1,\ 0 < y_1 y_2 < 2 \\ 0 & \text{otherwise} \end{cases}$$

(End of the quiz)