Please write* *Your name: _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

Let $X = \text{Exp}(2)$ and $Y = \text{Exp}(3)$ be two independent exponential random variables with parameters 2 and 3.

(1) Find the joint probability density function $f(x, y)$.

(2) In the same setting, write the double integral for $\mathbb{P}(X < Y < 2 - X)$. You do not have to evaluate the integral, but set up the limits of integration carefully.

(more questions on the next page)

Consider X and Y given by the joint density

$$f(x,y) = \begin{cases} \frac{1}{2} & \text{if } \ 0 \le x \le y \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

(3) Find the marginal p.d.f. $f_X(x)$.

(4) Find the marginal p.d.f. $f_Y(y)$.

(more questions on the next page)

Suppose the joint density function of the random variable X_1 and X_2 is

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1, 0 < x_2 < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(5) Let $Y_1 = \dfrac{1}{X_1}$ and $Y_2 = X_1 X_2$. What is the joint density function of Y_1 and Y_2?

(End of the quiz)