answers

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

Solving 5 out of 6 problems will give you 10 points in this quiz.

Suppose the joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & \text{otherwise.} \end{cases}$$

(1) Find the covariance $\mathrm{Cov}(X,Y)$. Do not simplify your answer.

Answer: $\mathbb{E}X = \mathbb{E}Y = \frac{1}{3}+\frac{1}{4}=\frac{7}{12}$, $\mathbb{E}XY = \frac{1}{6}+\frac{1}{6}=\frac{1}{3}$, $\mathrm{Cov}(X,Y)=\frac{1}{3}-\left(\frac{7}{12}\right)^2$

After simplification, which was not required, this is $-\frac{1}{144}$

(2) In the situation form the previous page, find the correlation coefficient $\rho(X,Y)$. Do not simplify your answer.

Answer: $\mathbb{E}X^2 = \mathbb{E}Y^2 = \frac{1}{4}+\frac{1}{6}=\frac{5}{12}$, $\mathrm{Var}X = \mathrm{Var}Y = \frac{5}{12}-\left(\frac{7}{12}\right)^2$,

$\rho(X,Y) = \dfrac{\frac{1}{3}-\left(\frac{7}{12}\right)^2}{\frac{5}{12}-\left(\frac{7}{12}\right)^2}$ After simplification, which was not required, this is $-\frac{1}{11}$

(3) In the situation form the previous page, find the conditional expectation $\mathbb{E}(X|Y)$. Do not simplify your answer.

Answer: $\dfrac{1/3+Y/2}{1/2+Y}$

(4) If X,Y,Z are exponential random variables with $\lambda=2$, use moment generating functions to find $\mathbb{E}(X+Y+Z)^2$

Answer: $m(t)=\left(\dfrac{2}{2-t}\right)^3$, $m'(t)=\dfrac{2^3\cdot 3}{(2-t)^4}$, $m''(t)=\dfrac{2^3\cdot 3\cdot 4}{(2-t)^5}$, $\mathbb{E}(X+Y+Z)^2 = m''(0)=3$

(5) If we rolls two dice, and X is the sum, can you write a formula for its moment generating function $m_X(t)$? Hint: your formula may be long, but you do not need to simplify it.

Answer: $m_X(t) = \dfrac{1}{36}\left(e^t+e^{2t}+e^{3t}+e^{4t}+e^{5t}+e^{6t}\right)^2$

(6) Find the moment generating function $m_X(t)$ for a random variable X with the p.d.f. $f_X(x)=\dfrac{1}{9}$if $2<x<5$, $\dfrac{2}{9}$if $5<x<8$, 0 otherwise

Answer: $\dfrac{e^{5t}-e^{2t}}{9t}+2\dfrac{e^{8t}-e^{5t}}{9t}$