Suppose the joint density function of the random variables $X$ and $Y$ is
\[ f(x, y) = \begin{cases} 
  x + y & 0 < x < 1, \ 0 < y < 1 \\
  0 & \text{otherwise} 
\end{cases} \]

(1) Find the covariance $\text{Cov}(X, Y)$. Do not simplify your answer.

**Answer:** $E[X] = E[Y] = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$, $E[XY] = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$, $\text{Cov}(X, Y) = \frac{1}{3} - \left(\frac{7}{12}\right)^2$

After simplification, which was not required, this is $-\frac{1}{144}$

(2) In the situation form the previous page, find the correlation coefficient $\rho(X, Y)$.

Do not simplify your answer.

**Answer:** $E[X^2] = E[Y^2] = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$, $\text{Var}(X) = \text{Var}(Y) = \frac{5}{12} - \left(\frac{7}{12}\right)^2$,

\[
\rho(X, Y) = \frac{\frac{1}{3} - \left(\frac{7}{12}\right)^2}{\frac{5}{12} - \left(\frac{7}{12}\right)^2} \quad \text{After simplification, which was not required, this is } -\frac{1}{11}
\]

(3) In the situation form the previous page, find the conditional expectation $E(X|Y)$.

Do not simplify your answer.

**Answer:** $\frac{1/3 + Y/2}{1/2 + Y}$

(4) If $X, Y, Z$ are exponential random variables with $\lambda = 2$, use moment generating functions to find $E(X + Y + Z)^2$

**Answer:** $m(t) = \left(\frac{2}{2-t}\right)^3$, $m'(t) = \frac{2 \cdot 3}{(2-t)^4}$, $m''(t) = \frac{2 \cdot 3 \cdot 4}{(2-t)^5}$, $E(X + Y + Z)^2 = m''(0) = 3$

(5) If we rolls two dice, and $X$ is the sum, can you write a formula for its moment generating function $m_X(t)$? Hint: your formula may be long, but you do not need to simplify it.

**Answer:** $m_X(t) = \frac{1}{36} (e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t})^2$

(6) Find the moment generating function $m_X(t)$ for a random variable $X$ with the p.d.f. $f_X(x) = \frac{1}{9}$ if $2 < x < 5$, $\frac{2}{9}$ if $5 < x < 8$, $0$ otherwise

**Answer:** $\frac{e^{5t} - e^{2t}}{9t} + 2\frac{e^{8t} - e^{5t}}{9t}$