Please write ***Your name:*** ____________________

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed. Hint: use $\Phi(x)$ for the $\mathcal{N}(0,1)$ distribution function: $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{x} e^{-y^2/2}dy=\mathbb{P}(Z<x)$ where Z is the standard normal.

Solving 5 out of 6 problems will give you 10 points in this quiz.

(1) Let $X_1, X_2, \ldots, X_k$ be independent exponential random variables with parameter $\lambda=3$. Use the Central Limit Theorem to approximate $\mathbb{P}\left(\sum_{i=1}^{k} X_i > a\right)$. Your answer should contain Φ, k, a, fractions, but should not contain symbols μ, σ.

Answer: $\mathbb{P}\left(\sum_{i=1}^{k} X_i > a\right) \approx 1-\Phi\left(\frac{3a-k}{\sqrt{k}}\right)$

(2) For which a we have $\mathbb{P}\left(\sum_{i=1}^{25} X_i > a\right) \approx 1-\Phi(4)$? ***Answer:*** $a=15$

(3) For which a we have $\mathbb{P}\left(\sum_{i=1}^{9} X_i > a\right) \approx \Phi(1)$? ***Answer:*** $a=2$

If the joint density function of the random variables X and Y is

$$f(x,y)=\begin{cases} ax^2y^2 & 0<x<1,\ 0<y<1 \\ 0 & \text{otherwise.} \end{cases}$$

(4) find a ***Answer:*** $a=9$

(5) find the covariance $\mathrm{Cov}(X,Y)$ ***Answer:*** $\mathrm{Cov}(X,Y)=0$ because X and Y are independent.
(hint: this is an easy question and you can find the answer without computing any integrals)

(6) find the conditional expectation $\mathbb{E}(X|Y)$

Answer: because X and Y are independent, $\mathbb{E}(X|Y)=\mathbb{E}(X)=\frac{3}{4}$