Please write Your name:

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed. Hint: use $\Phi(x)$ for the $\mathcal{N}(0,1)$ distribution function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2}dy = P(Z < x)$ where $Z$ is the standard normal.

Solving 5 out of 6 problems will give you 10 points in this quiz.

(1) Let $X_1, X_2, \ldots, X_k$ be independent exponential random variables with parameter $\lambda = 3$. Use the Central Limit Theorem to approximate $P\left(\sum_{i=1}^{k} X_i > a\right)$. Your answer should contain $\Phi$, $k$, $a$, fractions, but should not contain symbols $\mu, \sigma$.

(2) For which $a$ do we have $P\left(\sum_{i=1}^{25} X_i > a\right) \approx 1 - \Phi(4)$?

(3) For which $a$ do we have $P\left(\sum_{i=1}^{9} X_i > a\right) \approx \Phi(1)$?

(more questions on the next page)
If the joint density function of the random variables $X$ and $Y$ is

$$f(x, y) = \begin{cases} \alpha x^2 y^2 & 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(4) find $\alpha$

(5) find the covariance $\text{Cov}(X, Y)$

*(hint: this is an easy question and you can find the answer without computing any integrals)*

(6) find the conditional expectation $\mathbb{E}(X|Y)$