

Please write **Your name:** \_\_\_\_\_

**Show all work.** You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed. Hint: use  $\Phi(x)$  for the  $\mathcal{N}(0, 1)$  distribution function:  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-y^2/2} dy = \mathbb{P}(Z < x)$  where  $Z$  is the standard normal.

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 Solving 5 out of 6 problems will give you 10 points in this quiz.  
 .....

(1) Let  $X_1, X_2, \dots, X_k$  be independent exponential random variables with parameter  $\lambda = 3$ . Use the Central Limit Theorem to approximate  $\mathbb{P}\left(\sum_{i=1}^k X_i > a\right)$ . Your answer should contain  $\Phi$ ,  $k$ ,  $a$ , fractions, but should not contain symbols  $\mu, \sigma$ .

(2) For which  $a$  we have  $\mathbb{P}\left(\sum_{i=1}^{25} X_i > a\right) \approx 1 - \Phi(4)$ ?

(3) For which  $a$  we have  $\mathbb{P}\left(\sum_{i=1}^9 X_i > a\right) \approx \Phi(1)$ ?

If the joint density function of the random variables  $X$  and  $Y$  is

$$f(x, y) = \begin{cases} ax^2y^2 & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(4) find  $a$

(5) find the covariance  $\text{Cov}(X, Y)$

*(hint: this is an easy question and you can find the answer without computing any integrals)*

(6) find the conditional expectation  $\mathbb{E}(X|Y)$

(End of the quiz)