1. Suppose there are 7 black pens and 5 red pens. How many ways can we choose 4 pens?

**Answer:** \( \binom{12}{4} \) because there are 7 + 5 = 12 pens in total.

Note that \( \binom{12}{4} = 495 \) but this answer was not necessary for full credit.

2. Suppose there are 7 black pens and 5 red pens. How many ways can we choose 4 pens if we need two of each color?

**Answer:** \( \binom{7}{2} \cdot \binom{5}{2} \) because each color is chosen independently and so we have to multiply the number of choices for each color.

Note that \( \binom{7}{2} \cdot \binom{5}{2} = 210 \) but this answer was not necessary for full credit.

3. Suppose there are 7 black pens and 5 red pens. How many ways can we choose 4 pens if we need at least one of each color?

**Answer:** \( \binom{7}{1} \cdot \binom{5}{3} + \binom{7}{2} \cdot \binom{5}{2} + \binom{7}{3} \cdot \binom{5}{1} \) because we can choose one, two, or three black pens, and the numbers for each of these choices need to be added. An alternative correct solution is \( \binom{12}{4} - \binom{5}{4} - \binom{7}{4} \) because we can take the answer from question (1) and subtract that number of choices when all pens are black or all pens are red.

Note that \( \binom{7}{1} \cdot \binom{5}{3} + \binom{7}{2} \cdot \binom{5}{2} + \binom{7}{3} \cdot \binom{5}{1} = 70+210+175 = \binom{12}{4} - \binom{7}{4} - \binom{5}{4} = 495-35-5 = 455 \) but this answer was not necessary for full credit.

4. If a student council contains 11 people, how many ways are there to elect a president, a vice president, and a 4 person committee?

**Answer:** \( 11 \cdot 10 \cdot \binom{9}{4} \) because there are 11 choices for the president, and after that there will be 10 independent choices for the vice president, and the committee of 4 is chosen from the remaining 9 people independently.

Note that \( 11 \cdot 10 \cdot \binom{9}{4} = 13860 \) but this answer was not necessary for full credit.

(End of the quiz)