Please write **_Your name:_** ____________________

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

(1) Suppose there are 7 black pens and 5 red pens. How many ways can we choose 4 pens?

Answer: $\binom{12}{4}$ because there are $7+5=12$ pens in total.

Note that $\binom{12}{4}=495$ but this answer was not necessary for full credit.

(2) Suppose there are 7 black pens and 5 red pens. How many ways can we choose 4 pens if we need two of each color?

Answer: $\binom{7}{2}\cdot\binom{5}{2}$ because each color is chosen independently and so we have to multiply the number of choices for each color.

Note that $\binom{7}{2}\cdot\binom{5}{2}=210$ but this answer was not necessary for full credit.

(3) Suppose there are 7 black pens and 5 red pens. How many ways can we choose 4 pens if we need at least one of each color?

Answer: $\binom{7}{1}\cdot\binom{5}{3}+\binom{7}{2}\cdot\binom{5}{2}+\binom{7}{3}\cdot\binom{5}{1}$ because we can choose one, two, or three black pens, and the numbers for each of these choices need to be added. An alternative correct solution is $\binom{12}{4}-\binom{5}{4}-\binom{7}{4}$ because we can take the answer from question (1) and subtract that number of choices when all pens are black or all pens are red.

Note that $\binom{7}{1}\cdot\binom{5}{3}+\binom{7}{2}\cdot\binom{5}{2}+\binom{7}{3}\cdot\binom{5}{1}=70+210+175=\binom{12}{4}-\binom{7}{4}-\binom{5}{4}=495-35-5=455$ but this answer was not necessary for full credit.

(4) If a student council contains 11 people, how many ways are there to elect a president, a vice president, and a 4 person committee?

Answer: $11\cdot 10\cdot\binom{9}{4}$ because there are 11 choices for the president, and after that there will be 10 independent choices for the vice president, and the committee of 4 is chosen from the remaining 9 people independently.

Note that $11\cdot 10\cdot\binom{9}{4}=13860$ but this answer was not necessary for full credit.

(End of the quiz)