Please write* *Your name: _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed. One two sided page of handwritten notes is allowed.

. . . . . .

(1a) A password must contain exactly 1 capital letter, 3 lowercase letters, and 3 digits. Characters can be repeated, and the order matters. How many different 7-character passwords are possible?
Answer: $\binom{7}{1} \cdot \binom{6}{3} \cdot 26^4 \cdot 10^3$

(1b) Answer the same question if all the characters must be different. Here B and b are considered different characters. ***Answer:*** $\binom{7}{1} \cdot \binom{6}{3} \cdot 26^2 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8$

(2a) What is the probability that if we roll 3 dice, that all three are equal? ***Answer:*** $\frac{1}{36}$

(2b) What is the probability that if we roll 3 dice, that all three are different? ***Answer:*** $\frac{5}{6} \cdot \frac{4}{6} = \frac{20}{36} = \frac{5}{9}$

(2c) What is the probability that if we roll 3 dice, that two of them are equal and one is different from the other two? In this question the order does not matter. ***Answer:*** $\frac{5}{6} \cdot \frac{1}{6} \cdot \binom{3}{2} = \frac{15}{36} = \frac{5}{12}$

(2d) What is the sum of the probabilities in questions (2a), (2b) and (2c)? Explain
Answer: 1, because it is the total probability

(2e) What is the probability that if we roll 3 dice, the sum is 5? ***Answer:*** $2 \cdot \left(\frac{1}{6}\right)^3 \cdot \binom{3}{2} = \frac{1}{36}$

Suppose that X is a random variable with the outcomes $\{0, 1, 2, 3\}$. The corresponding probabilities are given by

$$\mathbb{P}(X = 0) = \frac{1}{3}, \qquad \mathbb{P}(X = 1) = \frac{1}{6}, \qquad \mathbb{P}(X = 2) = \frac{1}{6}, \qquad \mathbb{P}(X = 3) = \frac{1}{3}$$

(3a) Make a picture for the probability mass function. Indicate on this picture the location of $\mathbb{E}X$ ***Answer:*** *will be explained in class*

(3b) Compute $\mathbb{E}X$. ***Answer:*** $\mathbb{E}X = 3/2$

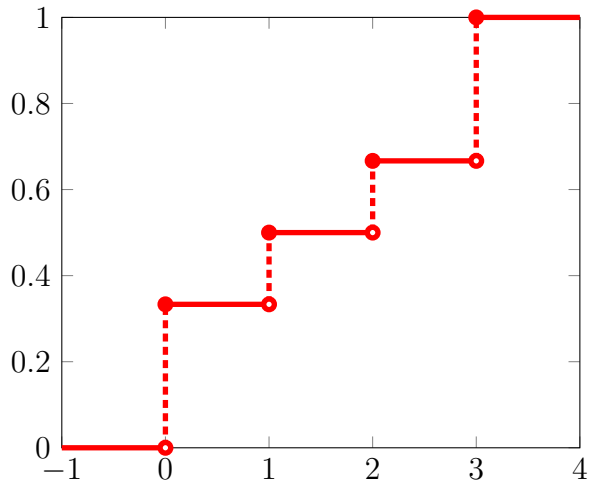
(3c) Does your answer for $\mathbb{E}X$ agree with your picture above? ***Answer:*** Yes

(3d) Find $\mathrm{Var}(X)$ but do not simplify. ***Answer:*** $\mathrm{Var}(X) = \frac{1}{6} + 4 \cdot \frac{1}{6}, +9 \cdot \frac{1}{3} - (3/2)^2 = \frac{19}{12}$

(3e) Find the cumulative distribution function F_X of X using the cases provided below. ***Answer:***

$$F_X(x) = \begin{cases} 0, & \text{for } -\infty < x < 0 \\ 1/3 & \text{for } 0 \leqslant x < 1 \\ 1/2 & \text{for } 1 \leqslant x < 2 \\ 2/3 & \text{for } 2 \leqslant x < 3 \\ 1 & \text{for } 3 \leqslant x < \infty \end{cases}$$

(3f) Plot the cumulative distribution function F_X of X using the chart provided below. Accurately label values at x and y axes. ***Answer:***



[(*optional questions for extra credit*)]:

You roll a dice 4 times and win \$1 for every even number rolled, and \$2 for every odd number rolled. What is the probability that you win \$7 or more? ***Answer:*** $\frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

What is the expected number of dollars that you win if you play this game? ***Answer:*** $\mathbb{E}X = 6$

What is the standard deviation of the number of dollars that you win if you play this game? ***Answer:*** Write $X = X_1 + X_2 + X_3 + X_4$, which is a sum of independent random variables. Then $\mathrm{Var}(X) = 4\mathrm{Var}(X_1) = 1$ and so $\mathrm{SD}(X) = 1$

— end of the test —