(1a) A password must contain exactly 1 capital letter, 3 lowercase letters, and 3 digits. Characters can be repeated, and the order matters. How many different 7-character passwords are possible?

Answer: \[ \binom{7}{1} \cdot \binom{6}{3} \cdot 26^4 \cdot 10^3 \]

(1b) Answer the same question if all the characters must be different. Here $B$ and $b$ are considered different characters. Answer: \[ \binom{7}{1} \cdot \binom{6}{3} \cdot 26^2 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \]

(2a) What is the probability that if we roll 3 dice, that all three are equal? Answer: \( \frac{1}{36} \)

(2b) What is the probability that if we roll 3 dice, that all three are different? Answer: \( \frac{5}{6} \cdot \frac{4}{6} = \frac{20}{36} = \frac{5}{9} \)

(2c) What is the probability that if we roll 3 dice, that two of them are equal and one is different from the other two? In this question the order does not matter. Answer: \( \frac{5}{6} \cdot \frac{1}{6} \cdot \binom{3}{2} = \frac{15}{36} = \frac{5}{12} \)

(2d) What is the sum of the probabilities in questions (2a), (2b) and (2c)? Explain

Answer: 1, because it is the total probability

(2e) What is the probability that if we roll 3 dice, the sum is 5? Answer: \( 2 \cdot \left( \frac{1}{6} \right)^3 \cdot \binom{3}{2} = \frac{1}{36} \)
Suppose that $X$ is a random variable with the outcomes \{0, 1, 2, 3\}. The corresponding probabilities are given by
\[ P(X = 0) = \frac{1}{3}, \quad P(X = 1) = \frac{1}{6}, \quad P(X = 2) = \frac{1}{6}, \quad P(X = 3) = \frac{1}{3}. \]

(3a) Make a picture for the probability mass function. Indicate on this picture the location of $\mathbb{E}X$. Answer: will be explained in class

(3b) Compute $\mathbb{E}X$. Answer: $\mathbb{E}X = 3/2$

(3c) Does your answer for $\mathbb{E}X$ agree with your picture above? Answer: Yes

(3d) Find $\text{Var}(X)$ but do not simplify. Answer: $\text{Var}(X) = \frac{1}{6} + 4 \cdot \frac{1}{6} + 9 \cdot \frac{1}{3} - (3/2)^2 = \frac{19}{12}$

(3e) Find the cumulative distribution function $F_X$ of $X$ using the cases provided below. Answer:
\[
F_X(x) = \begin{cases} 
0, & \text{for } -\infty < x < 0 \\
1/3, & \text{for } 0 \leq x < 1 \\
1/2, & \text{for } 1 \leq x < 2 \\
2/3, & \text{for } 2 \leq x < 3 \\
1, & \text{for } 3 \leq x < \infty 
\end{cases}
\]

(3f) Plot the cumulative distribution function $F_X$ of $X$ using the chart provided below. Accurately label values at $x$ and $y$ axes. Answer:

[(optional questions for extra credit)]:

You roll a dice 4 times and win $1 for every even number rolled, and $2 for every odd number rolled. What is the probability that you win $7 or more? Answer: $\frac{1}{16} + \frac{4}{16} = \frac{5}{16}$

What is the expected number of dollars that you win if you play this game? Answer: $\mathbb{E}X = 6$

What is the standard deviation of the number of dollars that you win if you play this game? Answer: Write $X = X_1 + X_2 + X_3 + X_4$, which is a sum of independent random variables. Then $\text{Var}(X) = 4\text{Var}(X_1) = 1$ and so $\text{SD}(X) = 1$

— end of the test —