

Please write **Your name:** _____

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

(1a) Suppose that X is a random variable with the outcomes $\{1, 2, 3\}$. The corresponding probabilities are given by

$$\mathbb{P}(X = 1) = \frac{2}{5}, \quad \mathbb{P}(X = 2) = \frac{1}{5}, \quad \mathbb{P}(X = 3) = \frac{2}{5}$$

Find its expected value $\mathbb{E}X$ and $\mathbb{E}X^2$. **Answer:**

$$\mathbb{E}X = 1 \cdot \frac{2}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{2}{5} = \frac{10}{5} = 2 \quad \text{and}$$

$$\mathbb{E}X^2 = 1^2 \cdot \frac{2}{5} + 2^2 \cdot \frac{1}{5} + 3^2 \cdot \frac{2}{5} = \frac{24}{5}$$

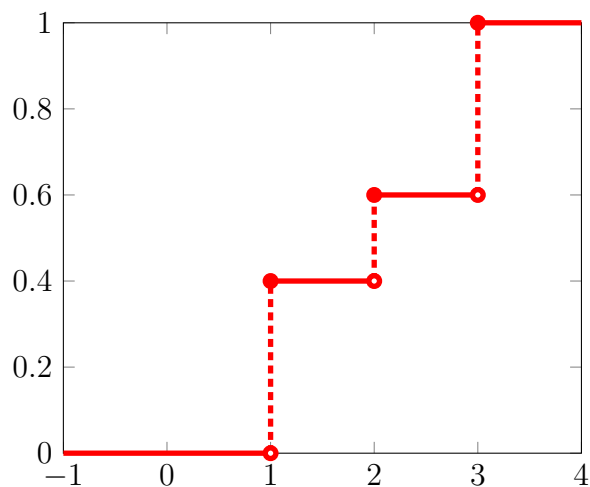
(1b) Find the variance $\text{Var}(X)$. **Answer:** $\text{Var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2 = \frac{24}{5} - (2)^2 = \frac{4}{5}$

(2a) Suppose that X is a random variable with the outcomes $\{1, 2, 3\}$. The corresponding probabilities are given as in question. Find the cumulative distribution function F_X of X using the cases provided below.

Answer:

$$F_X(x) = \begin{cases} 0 & \text{for } -\infty < x < 1 \\ \frac{2}{5} & \text{for } 1 \leq x < 2 \\ \frac{3}{5} & \text{for } 2 \leq x < 3 \\ 1 & \text{for } 3 \leq x < \infty \end{cases}$$

(2b) Plot the cumulative distribution function F_X of X using the chart provided below. Accurately label values at x and y axes. **Answer:**



(3a) Toss two fair coins and define the random variable X as the number of heads. Thus, X can have values 0, 1, or 2. Find the probability mass function of X .

Answer: $\mathbb{P}(X = 0) = \frac{1}{4}, \quad \mathbb{P}(X = 1) = \frac{1}{2}, \quad \mathbb{P}(X = 2) = \frac{1}{4}$

(3b) Find the variance of X from the previous question. **Answer:** $\text{Var}(X) = \frac{1}{2}$, three different solutions will be explained in class.