Please write Your name: ______________________________

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

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In questions on this page, we discuss a gene mutation which can occur in an individual with probability $p = 10^{-2} = 0.01$, which is one percent.

(1) Find the probability that a random group of 50 people have nobody with this particular gene mutation.
Answer: The binomial distribution gives the answer $.99^{50} \approx .6050$
while the Poisson approximation gives the answer $e^{-1/2} \approx 0.6065$
Hence a good approximate answer is probability 60.5%

(2) Find the probability that a random group of 50 people have at least two people with this particular gene mutation.
Answer: The binomial distribution gives the answer $1 - .99^{50} - 50 \cdot .01 \cdot .99^{49} \approx .0894$
while the Poisson approximation gives the answer $1 - e^{-1/2} - e^{-1/2}/2 \approx 0.0902$
Hence a good approximate answer is probability 9%.

(3) Suppose X is a Poisson random variable. If $\mathbb{P}(X = 0) = \frac{1}{3}$, can you find $Var(X)$?
Answer: $Var(X) = \lambda = \ln(3)$

(4) Each day a student wakes up and flips three fair coins. If all three coins are heads, then the student goes to the swimming pool. Otherwise the student goes to the gym. Let X be the number of days until the student will go to the swimming pool. Find $\mathbb{E}X$ and $\mathrm{Var}(X)$.
Answer: This is a geometric random variable with $p = 1/8$, $EX = 1/p = 8$, $Var(X) = \frac{1-p}{p^2} = 56$

(5) Suppose X is a Binomial random variable with $\mathbb{E}X = 6$ and $\mathrm{Var}(X) = 4$. Can you find p and n?
Answer: $\mathbb{E}X = 6 = np$ and $\mathrm{Var}(X) = 4 = np(1-p)$ and so $1 - p = Var(X)/EX = 2/3$. Hence $p = 1/3$ and $n = 18$

[(*optional question for extra credit*)]:
Does there exist a Binomial random variable X with $\mathbb{E}X = 3$ and $\mathrm{Var}(X) = 6$? Explain.
Answer: this is not possible because $0 < p < 1$ and so $0 < 1 - p < 1$ and therefore $0 < Var(X) < \mathbb{E}X$ for any Binomial random variable X. If we try to solve like in the previous question, we would get $\mathbb{E}X = 3 = np$ and $\mathrm{Var}(X) = 6 = np(1-p)$ and so $1-p = Var(X)/EX = 2$ which would require $p = -1$, which is impossible.