Please write Your name:

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

In questions on this page, we discuss a gene mutation which can occur in an individual with probability $p = 10^{-2} = 0.01$, which is one percent.

(1) Find the probability that a random group of 50 people have nobody with this particular gene mutation.

(2) Find the probability that a random group of 50 people have at least two people with this particular gene mutation.

(3) Suppose $X$ is a Poisson random variable. If $P(X = 0) = \frac{1}{3}$, can you find $Var(X)$?

(More on the back)
(4) Each day a student wakes up and flips three fair coins. If all three coins are heads, then the student goes to the swimming pool. Otherwise the student goes to the gym. Let $X$ be the number of days until the student will go to the swimming pool. Find $\mathbb{E}X$ and $\text{Var}(X)$.

(5) Suppose $X$ is a Binomial random variable with $\mathbb{E}X = 6$ and $\text{Var}(X) = 4$. Can you find $p$ and $n$?

[[optional question for extra credit]]:
Does there exist a Binomial random variable $X$ with $\mathbb{E}X = 3$ and $\text{Var}(X) = 6$? Explain.