

Please write **Your name:** \_\_\_\_\_

**Show all work.** You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

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In this quiz we discuss a random variable  $X$  with a probability density function  $f(x)$  which is given by  $f(x) = a(x + 1)$  when  $-1 < x < 1$ , and  $f(x) = 0$  for all other  $x$ . Here  $a$  is a number.

(1) Find  $a$ .

$$a = 1/2 \text{ because } \int_{-1}^1 (1 + x)dx = (x + x^2/2)|_{x=-1}^{x=1} = 2$$

(2) Find  $\mathbb{P}(X < 0)$ .

$$\mathbb{P}(X < 0) = \int_{-1}^0 \frac{1}{2}(1 + x)dx = \frac{1}{2}(x + x^2/2)|_{x=-1}^0 = \frac{1}{4}$$

(3) Find  $\mathbb{E}X$ .

$$\mathbb{E}X = \int_{-1}^1 \frac{1}{2}x(1 + x)dx = \frac{1}{2}(x^2/2 + x^3/3)|_{x=-1}^{x=1} = \frac{1}{3}$$

(4) Find  $\mathbb{E}X^2$ .

$$\mathbb{E}X^2 = \int_{-1}^1 \frac{1}{2}x^2(1 + x)dx = \frac{1}{2}(x^3/3 + x^4/4)|_{x=-1}^{x=1} = \frac{1}{3}$$

(5) Find the cumulative distribution function  $F(x)$  using the cases provided below.

$$F_X(x) = \begin{cases} 0 & \text{for } -\infty < x < -1 \\ \frac{1}{2}(x + x^2/2 + 1/2) & \text{for } -1 < x < 1 \\ 1 & \text{for } 1 < x < \infty \end{cases}$$

Here  $\frac{1}{2}(x + x^2/2 + 1/2) = \frac{1}{2}(x + x^2/2)|_{x=-1}^x$

[(optional question for extra credit)]:

Plot the probability density function  $f(x)$  and the cumulative distribution function  $F(x)$  using the charts provided below. Accurately label values at  $x$  and  $y$  axes.

