

**Sample Test 2 questions:**

- (1) Let  $X$  be a Poisson random variable with  $\mathbb{E}X = 4$ . Find the formula for  $\mathbb{P}(2 \leq X \leq 5)$ .
- (2) Let  $X$  be a binomial random variable with  $\mathbb{E}X = 4$  and  $n = 10$ . Find the formula for  $\mathbb{P}(2 \leq X \leq 5)$ .
- (3) Let  $X$  be the number of coin tosses until we have one head. Find  $\mathbb{E}X$  and  $\text{Var}X$ . Find  $\mathbb{P}(X \geq 3)$

(4) Suppose  $X$  has the following p.d.f.

$$f(x) = \begin{cases} ax^2 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find  $a$ ,  $\mathbb{E}X$ ,  $\text{Var}X$ , and the cdf  $F(x)$ . Make a picture of the p.d.f. and c.d.f.

(5) Suppose  $X$  has the following p.d.f.

$$f(x) = \begin{cases} \frac{a}{x^2} & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find  $a$ ,  $\mathbb{E}X$ ,  $\text{Var}X$ , and the cdf  $F(x)$ . Make a picture of the p.d.f. and c.d.f.

- (6) Find the numerical value for  $\mathbb{P}(-2 \leq X \leq 3)$  if  $X$  is  $\mathcal{N}(3, 4)$ . Your answer should include  $\Phi$  twice. After that, use the normal table attached in the end of the quiz find the approximate answer.
- (7) Find a formula for  $\mathbb{P}(-2 \leq X \leq 3)$  if  $X$  is  $\mathcal{N}(-3, 4)$ . Your answer should include  $\Phi$  twice. After that, use the normal table attached in the end of the quiz find the approximate answer.
- (8) If  $Z_1$  and  $Z_2$  are standard normal independent random variables, and  $X = 3Z_1 + 4Z_2$ , find  $\mathbb{P}(-2 \leq X \leq 3)$ . Your answer should include  $\Phi$  twice. After that, use the normal table attached in the end of the quiz find the approximate answer.

(9) Let  $X$  be a binomial random variable with  $\mathbb{E}X = 60$  and  $n = 150$ . Find the normal approximation for  $\mathbb{P}(54 \leq X \leq 72)$ .

(10) Let  $X$  be the number of tails in 25 fair coin tosses. Find the best normal approximation for  $\mathbb{P}(X \leq 15)$ .

(11) Let a fair dice be thrown 2000 times. Find the normal approximation of the probability that 6 appears at least 300 times.

(12) Let  $X$  be an exponential random variable with parameter  $\lambda = 2$ . Find

- (a) Find  $P(X > 3)$
- (b) Find  $P(X > 3|P > 2)$
- (c) Find  $P(4 < X < 6|P > 2)$

(13) Let  $X$  be a uniform random variable on  $[1, 5]$ . Find

- (a) Find  $P(X > 3)$
- (b) Find  $P(X > 3|P > 2)$
- (c) Find  $P(4 < X < 6|P > 2)$
- (d) Find  $\mathbb{E}e^{2X}$ .
- (e) Find  $\mathbb{E}\frac{1}{X^2}$ .

(14) Consider random variables  $X$  and  $Y$  with the joint probability density function

$$f(x, y) = \begin{cases} axy^2 & \text{if } 0 \leq x \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find  $a$
- (b) Find the marginal p.d.f.  $f_X(x)$ .
- (c) Find the marginal p.d.f.  $f_Y(y)$ .

(15) Consider random variables  $X$  and  $Y$  with the joint probability density function

$$f(x, y) = \begin{cases} axy^2 & \text{if } 0 \leq y \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find  $a$
- (b) Find the marginal p.d.f.  $f_X(x)$ .
- (c) Find the marginal p.d.f.  $f_Y(y)$ .

(16) Consider random variables  $X$  and  $Y$  with the joint probability density function

$$f(x, y) = \begin{cases} axy^2 & \text{if } 0 \leq x, 0 \leq y, x + y < 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find  $a$
- (b) Find the marginal p.d.f.  $f_X(x)$ .
- (c) Find the marginal p.d.f.  $f_Y(y)$ .

(17) In the previous problem, let  $Y_1 = X + Y$  and  $Y_2 = X - Y$ . Find the joint probability density function  $f_{Y_1, Y_2}(y_1, y_2)$