Please write Your name:

Show all work. You should either write at a sentence explaining your reasoning, or annotate your math work with brief explanations. There is no need to simplify, and no calculators are needed.

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(1) Let X be the time that a car can run until a major repair. If $\mathbb{E}X = 2$, and X is exponentially distributed, what is $\mathbb{P}(2 < X < 5)$?

(2) Given that this car has ran 2 years without a repair, what is the conditional probability that it will run 3 more years without a major repair?

(3) What is the probability density function of $Y = X^2$?

(4) Find Var(X) if X is uniformly distributed on the interval [-1, 5]. Show all steps.

[(optional questions for extra credit)]: Let a, b, c be positive numbers, b > 1, and the probability density function f(x) of a random variable X be defined by $f(x) = ax^{-b}$ for x > c and f(x) = 0 for $x \le c$.

• What is the relation between a, b, c?

• What is the necessary and sufficient condition for b so that $\mathbb{E}X < +\infty$?

• What is the necessary and sufficient condition for b so that $Var(X) < +\infty$?