Consider random variables $X$ and $Y$ given by the joint density
\[ f(x, y) = \begin{cases} 
  x + y & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\
  0 & \text{otherwise.}
\end{cases} \]

Find $\text{Cov}(X, Y)$  \textbf{Answer:} \( \frac{1}{3} - \frac{7}{12} \cdot \frac{7}{12} = -\frac{1}{144} \)

In the same situation, find $\mathbb{E}(X|Y)$.
\textbf{Answer:} \( \mathbb{E}(X|Y) = \frac{1/3 + Y/2}{1/2 + Y} \)

\[(\text{optional question for extra credit)}]: \text{If } Z_1, Z_2 \text{ are independent standard normal random variables, and } X = 3Z_1 + 4Z_2, Y = 3Z_1 - 4Z_2, \text{ find } \rho(X, Y). \text{ Do not use any integrals or derivatives.}\]
\textbf{Answer:} \text{Note that } Z_1 \text{ and } Z_2 \text{ are independent, but } X \text{ and } Y \text{ are not independent.}
\[\mathbb{E}XY = \mathbb{E}(3Z_1 + 4Z_2)(3Z_1 - 4Z_2) = \mathbb{E}(9Z_1^2 - 16Z_2^2) = 9 - 16 = -7,\]
\[\text{Var}X = \text{Var}Y = 9 + 16 = 25,\]
therefore the correlation coefficient is $\rho(X, Y) = -\frac{7}{25}$. 