

Please write **Your name:** \_\_\_\_\_

**Show all work:** either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are allowed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions.

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(1) Suppose we roll two dice, and consider events  $A = \{\text{the first die is a 5}\}$ ,  $B = \{\text{the sum is 10}\}$ . Are these two events independent? Explain.

**Answer:**  $\mathbb{P}(A) = 1/6$ ,  $\mathbb{P}(B) = 1/12$ ,  $\mathbb{P}(A \cap B) = 1/36$ , therefore  $A$  and  $B$  not independent.

(2) In the same situation, compute  $\mathbb{P}(A | B)$  and  $\mathbb{P}(B | A)$ .

**Answer:**  $\mathbb{P}(A | B) = 1/3$ ,  $\mathbb{P}(B | A) = 1/6$

Suppose that

- a flu test indicates the presence of the flu  $\frac{4}{5}$  of the times when the patient actually has the flu (this is called the true positive rate);
- the same test indicates the absence of flu  $\frac{4}{5}$  of the times when the patient actually does not have the flu (this is called the true negative rate);
- currently  $\frac{1}{4}$  of the population has the flu.

(3) For a random person, what is the probability that the flu test is positive?

**Answer:**  $\frac{4}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{3}{4} = \frac{7}{20}$

(4) Calculate the probability that a random person actually has the flu, given that the flu test is positive.

**Answer:**  $\frac{\frac{4}{5} \cdot \frac{1}{4}}{\frac{7}{20}} = \frac{4}{7}$