(1a) Suppose that X is a random variable with the outcomes $\{-1, 0, 1, 2\}$. The corresponding probabilities are given by

$$\mathbb{P}(X = -1) = \frac{1}{4}, \qquad \mathbb{P}(X = 0) = \frac{1}{4}, \qquad \mathbb{P}(X = 1) = \frac{1}{4}, \qquad \mathbb{P}(X = 2) = \frac{1}{4}$$

Find its expected value $\mathbb{E}X$ and $\mathbb{E}X^2$.

Answer: $\mathbb{E}X = \frac{1}{4}(-1 + 0 + 1 + 2) = \frac{1}{2}$, $\mathbb{E}X^2 = \frac{1}{4}((-1)^2 + 0^2 + 1^2 + 2^2) = \frac{3}{2}$.

(1b) Find the variance $\mathbf{Var}(X)$ and the standard deviation $SD(X)$

Answer: $\mathbf{Var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2 = 3/2 - 1/4 = 5/4$, $SD(X) = \frac{\sqrt{5}}{2}$

(2a) Suppose that X is a random variable with the outcomes $\{-1, 0, 1, 2\}$. The corresponding probabilities are given as in question (1) by

$$\mathbb{P}(X = -1) = \frac{1}{4}, \qquad \mathbb{P}(X = 0) = \frac{1}{4}, \qquad \mathbb{P}(X = 1) = \frac{1}{4}, \qquad \mathbb{P}(X = 2) = \frac{1}{4}$$

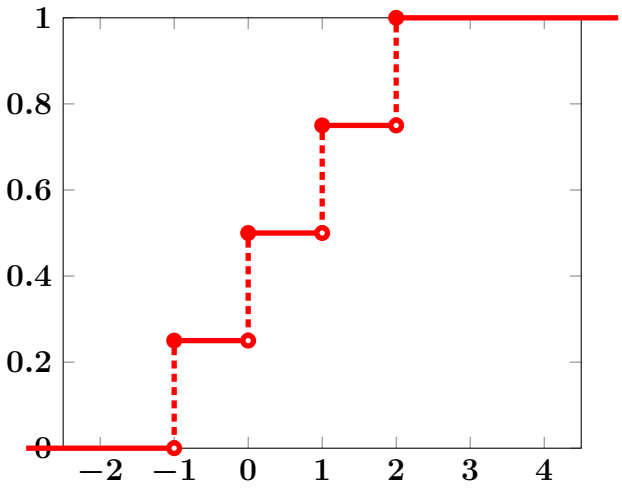
Find the cumulative distribution function F_X of X using the cases provided below.

Answer:

$$F_X(x) = \begin{cases} 0, & \text{for } -\infty < x < -1 \\ 1/4, & \text{for } -1 \leqslant x < 0 \\ 1/2, & \text{for } 0 \leqslant x < 1 \\ 3/4, & \text{for } 1 \leqslant x < 2 \\ 1, & \text{for } 2 \leqslant x < \infty \end{cases}$$

(2b) Plot the cumulative distribution function F_X of X using the chart provided below. Accurately label values at x and y axes.

Answer:



end of the quiz