Please write ***Your name:*** _____

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are allowed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions.

. . . . . .

(1a) Suppose that X is a random variable with the probability density given by $\boldsymbol{f(x) = a(2x - x^2)}$ when $\boldsymbol{0 < x < 2}$ and zero otherwise. Find the value of $\boldsymbol{a}$.

Answer: $\boldsymbol{a = 1/(\int_0^2 (2x - x^2)dx) = 3/4}$

1(b) Find the cumulative distribution function $\boldsymbol{F_X}$ of $\boldsymbol{X}$ using the cases provided below.

Answer:

$$\boldsymbol{F_X(x)} = \begin{cases} \boldsymbol{0}, & \text{for } \boldsymbol{-\infty < x < 0} \\ \boldsymbol{3x^2/4 - x^3/4} & \text{for } \boldsymbol{0 \leq x < 2} \\ \boldsymbol{1} & \text{for } \boldsymbol{2 \leq x < \infty} \end{cases}$$

(1c) Find its expected value $\mathbb{E}\boldsymbol{X}$.

Answer: $\boldsymbol{\mathbb{E}X = (3/4)\int_0^2 x(2x - x^2)dx = 1}$

(1d) Find the variance $\mathbf{Var}(\boldsymbol{X})$.

Answer: $\boldsymbol{\mathbf{Var}(X) = (3/4)\int_0^2 x^2(2x - x^2)dx - 1 = 1/5}$

end of the quiz