

Please write **Your name:** \_\_\_\_\_

**Show all work:** either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are allowed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions.

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In this quiz use the notation  $\Phi(x)$  for the distribution function for  $\mathcal{N}(0, 1)$ , that is

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-y^2/2} dy = \mathbb{P}(Z < x)$$

where  $Z$  is the standard normal random variable. You do not need a table of values of  $\Phi$ .

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- (1) We toss a pair of coins and say that we have a success if both are heads, and failure otherwise. Let  $S_n$  be the number of successes if we toss  $n$  pairs of coins. What is the mean and the standard deviation of  $S_n$  if  $n = 48$ ?

**Answer:**  $\mathbb{E}S_{48} = 12 \quad SD(S_{48}) = 3$

- (2) Estimate the probability that  $S_{48} \geq 15$  using the normal approximation. Do not use the continuity correction. Your answer should include  $\Phi$ .

**Answer:**  $\mathbb{P}(S_{48} \geq 15) \approx 1 - \Phi(1)$

- (3) In the same situation, estimate the probability that  $S_{48} = 12$  using the normal approximation with the continuity correction. Your answer should include  $\Phi$ .

**Answer:**  $\mathbb{P}(S_{48} = 12) \approx 2\Phi(1/6) - 1$

- (4) Find a formula for  $\mathbb{P}(-5 \leq X \leq 4)$  if  $X$  is  $\mathcal{N}(-2, 9)$ . Your answer should include  $\Phi$  twice.

**Answer:**  $\mathbb{P}(-5 \leq X \leq 4) = \Phi(2) + \Phi(1) - 1$

*end of the quiz*