Please write Your name:

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are allowed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions.

In this quiz use the notation \( \Phi(x) \) for the distribution function for \( \mathcal{N}(0, 1) \), that is

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)
\]

where \( Z \) is the standard normal random variable. You do not need a table of values of \( \Phi \).

(1) We toss a pair of coins and say that we have a success if both are heads, and failure otherwise. Let \( S_n \) be the number of successes if we toss \( n \) pairs of coins. What is the mean and the standard deviation of \( S_n \) if \( n = 48 \)?

Please write your answer here:

\[
\mathbb{E}S_{48} = \quad SD(S_{48}) =
\]

(2) Estimate the probability that \( S_{48} \geq 15 \) using the normal approximation. Do not use the continuity correction. Your answer should include \( \Phi \).

Please write your answer here:

\[
\mathbb{P}(S_{48} \geq 15) \approx
\]

Please go to the next page ...
(3) In the same situation, estimate the probability that $S_{48} = 12$ using the normal approximation with the continuity correction. Your answer should include $\Phi$.

Please write your answer here:

\[ \mathbb{P}(S_{48} = 12) \approx \]

(4) Find a formula for $\mathbb{P}(-5 \leq X \leq 4)$ if $X \sim \mathcal{N}(-2, 9)$. Your answer should include $\Phi$ twice.

Please write your answer here:

\[ \mathbb{P}(-5 \leq X \leq 4) = \]