(1) Car types $A, B, C$ are bought in numbers 100, 200, 300 respectively, and have accident rates 0.3, 0.2, 0.1 respectively. Given an accident, what is the probability that the car type $B$ is involved?

**Answer:** $P(B|\text{accident}) = \frac{40}{30 + 40 + 30} = \frac{2}{5}$

2(a) Suppose we toss 3 fair coins, and let $X$ be the number of heads. Find the probability mass function for $X$.  
**Answer:** $P(X = 0) = P(X = 3) = \frac{1}{8}$\quad $P(X = 1) = P(X = 2) = \frac{3}{8}$

2(b) Find $E_X$ and $\text{Var}(X)$.  
**Answer:** $E_X = \frac{3}{2}$\quad $\text{Var}(X) = \frac{3}{4}$

2(c) Suppose again we toss 3 fair coins, and let $X$ be the number of heads. Find the cumulative distribution function $F_X$ of $X$ using the cases provided below.

$$F_X(x) = \begin{cases} 
0, & \text{for } -\infty < x < 0 \\
\frac{1}{8}, & \text{for } 0 \leq x < 1 \\
\frac{1}{2}, & \text{for } 1 \leq x < 2 \\
\frac{7}{8}, & \text{for } 2 \leq x < 3 \\
1, & \text{for } 3 \leq x < \infty
\end{cases}$$

2(d) Plot the cumulative distribution function $F_X$ of $X$ using the chart provided below. Accurately label values at $x$ and $y$ axes.

(3a) Suppose we have 3 black and 3 red pens, and we select 2 pens in random. Let $A = \{\text{the first pen is red}\}$ and $B = \{\text{the second pen is red}\}$. Find if these events are independent.

**Answer:** $P(A) = \frac{1}{2}$\quad $P(B) = \frac{1}{2}$\quad $P(A \cap B) = \frac{1}{5}$\quad Are $A$ and $B$ independent? **no**

3(b) Find the probability the second pen is red, given that the first pen is red. **Answer:** $P(B|A) = \frac{2}{5}$

3(c) Suppose again that we have 3 black and 3 red pens, and we select 2 pens in random. Let $X$ be the number of red pens. Find the probability mass function for $X$.

**Answer:** $P(X = 0) = \frac{1}{5}$\quad $P(X = 1) = \frac{3}{5}$\quad $P(X = 2) = \frac{1}{5}$

3(d) Find $E_X$ and $\text{Var}(X)$. **Answer:** $E_X = 1$\quad $\text{Var}(X) = \frac{2}{5}$

*Please go to the next page ...*
Optional problem for extra credit. Suppose that currently 0.2 of population is infected with flu. We have a test with overall error rate $\alpha$, so that $\alpha$ is the false positive rate, and also is the false negative rate. Assume that if we administer this test to a random person, and it is positive, then the probability that this person has the flu is 0.8

What is $\alpha$? Answer: $\alpha = 1/17$ which solves \[
\frac{0.2(1 - \alpha)}{0.2(1 - \alpha) + 0.8\alpha} = 0.8
\]

end of the test