(1) Car types $\boldsymbol{A, B, C}$ are bought in numbers $\mathbf{100, 200, 300}$ respectively, and have accident rates $\mathbf{0.3, 0.2, 0.1}$ respectively. Given an accident, what is the probability that the car type $\boldsymbol{B}$ is involved?
Answer: $\mathbb{P}(\boldsymbol{B}|\boldsymbol{accident}) = \mathbf{40/(30+40+30) = 2/5}$

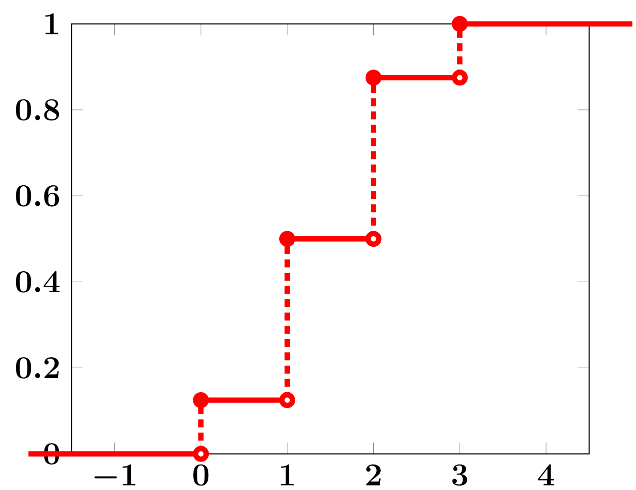
2(a) Suppose we toss 3 fair coins, and let $\boldsymbol{X}$ be the number of heads. Find the probability mass function for $\boldsymbol{X}$. ***Answer:*** $\mathbb{P}(\boldsymbol{X}=\mathbf{0}) = \mathbb{P}(\boldsymbol{X}=\mathbf{3}) = \mathbf{1/8}$ $\quad \mathbb{P}(\boldsymbol{X}=\mathbf{1}) = \mathbb{P}(\boldsymbol{X}=\mathbf{2}) = \mathbf{3/8}$

2(b) Find $\mathbb{E}\boldsymbol{X}$ and $\mathbf{Var}(\boldsymbol{X})$. ***Answer:*** $\mathbb{E}\boldsymbol{X} = \mathbf{3/2}$ $\quad \mathbf{Var}(\boldsymbol{X}) = \mathbf{3/4}$

(2c) Suppose again we toss 3 fair coins, and let $\boldsymbol{X}$ be the number of heads. Find the cumulative distribution function $\boldsymbol{F_X}$ of $\boldsymbol{X}$ using the cases provided below.

$$\boldsymbol{F_X(x)} = \begin{cases} \mathbf{0,} & \text{for } -\infty < \boldsymbol{x} < \mathbf{0} \\ \mathbf{1/8} & \text{for } \mathbf{0} \leqslant \boldsymbol{x} < \mathbf{1} \\ \mathbf{1/2} & \text{for } \mathbf{1} \leqslant \boldsymbol{x} < \mathbf{2} \\ \mathbf{7/8} & \text{for } \mathbf{2} \leqslant \boldsymbol{x} < \mathbf{3} \\ \mathbf{1} & \text{for } \mathbf{3} \leqslant \boldsymbol{x} < \infty \end{cases}$$

(2d) Plot the cumulative distribution function $\boldsymbol{F_X}$ of $\boldsymbol{X}$ using the chart provided below. Accurately label values at $\boldsymbol{x}$ and $\boldsymbol{y}$ axes.



(3a) Suppose we have 3 black and 3 red pens, and we select 2 pens in random.
Let $\boldsymbol{A} = \{\boldsymbol{the\ first\ pen\ is\ red}\}$ and $\boldsymbol{B} = \{\boldsymbol{the\ second\ pen\ is\ red}\}$.
Find if these events are independent.
Answer: $\mathbb{P}(\boldsymbol{A}) = \mathbf{1/2}$ $\mathbb{P}(\boldsymbol{B}) = \mathbf{1/2}$ $\mathbb{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \mathbf{1/5}$ Are $\boldsymbol{A}$ and $\boldsymbol{B}$ independent? ***no***

3(b) Find the probability the second pen is red, given that the first pen is red. ***Answer:*** $\mathbb{P}(\boldsymbol{B}|\boldsymbol{A}) = \mathbf{2/5}$

(3c) Suppose again that we have 3 black and 3 red pens, and we select 2 pens in random. Let $\boldsymbol{X}$ be the number of red pens. Find the probability mass function for $\boldsymbol{X}$.

Answer: $\mathbb{P}(\boldsymbol{X}=\mathbf{0}) = \mathbf{1/5}$ $\quad \mathbb{P}(\boldsymbol{X}=\mathbf{1}) = \mathbf{3/5}$ $\quad \mathbb{P}(\boldsymbol{X}=\mathbf{2}) = \mathbf{1/5}$

3(d) Find $\mathbb{E}\boldsymbol{X}$ and $\mathbf{Var}(\boldsymbol{X})$. ***Answer:*** $\mathbb{E}\boldsymbol{X} = \mathbf{1}$ $\mathbf{Var}(\boldsymbol{X}) = \mathbf{2/5}$

Please go to the next page ...

Optional problem for extra credit. Suppose that currently $\mathbf{0.2}$ of population is infected with flu. We have a test with overall error rate $\boldsymbol{\alpha}$, so that $\boldsymbol{\alpha}$ is the false positive rate, and also is the false negative rate. Assume that if we administer this test to a random person, and it is positive, then the probability that this person has the flu is $\mathbf{0.8}$

What is $\boldsymbol{\alpha}$? ***Answer:*** $\boldsymbol{\alpha = 1/17}$ which solves $\dfrac{\mathbf{0.2(1-\alpha)}}{\mathbf{0.2(1-\alpha)+0.8\alpha}} = \mathbf{0.8}$

end of the test