

Please write ***Your name:*** _____

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are allowed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions.

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- (1) Car types ***A, B, C*** are bought in numbers **100, 200, 300** respectively, and have accident rates **0.3, 0.2, 0.1** respectively. Given an accident, what is the probability that the car type ***B*** is involved?

Please write your answer here:

$\mathbb{P}(B|\text{accident}) =$

(more questions on the next page)

- 2(a) Suppose we toss 3 fair coins, and let $\mathbf{X}$ be the number of heads. Find the probability mass function for $\mathbf{X}$.

Please write your answer here:

$$\mathbb{P}(\mathbf{X} = 0) = \quad \mathbb{P}(\mathbf{X} = 1) = \quad \mathbb{P}(\mathbf{X} = 2) = \quad \mathbb{P}(\mathbf{X} = 3) =$$

- 2(b) Find $\mathbb{E}\mathbf{X}$ and $\text{Var}(\mathbf{X})$.

Please write your answer here:

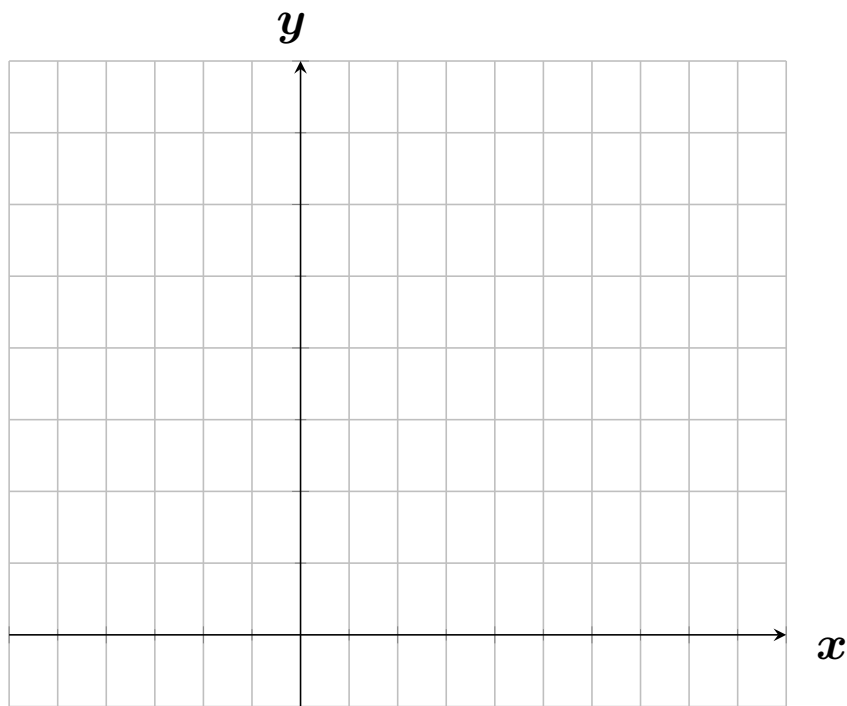
$$\mathbb{E}\mathbf{X} = \quad \text{Var}(\mathbf{X}) =$$

(more questions on the next page)

- (2c) Suppose again we toss 3 fair coins, and let $\mathbf{X}$ be the number of heads. Find the cumulative distribution function $\mathbf{F_X}$ of $\mathbf{X}$ using the cases provided below.

$$\mathbf{F_X}(x) = \begin{cases} 0, & \text{for } -\infty < x < \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} x \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} x \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} x \rule{1cm}{0.4pt} \\ \rule{1cm}{0.4pt}, & \text{for } \rule{1cm}{0.4pt} \leq x < \infty \end{cases}$$

- (2d) Plot the cumulative distribution function $\mathbf{F_X}$ of $\mathbf{X}$ using the chart provided below. Accurately label values at $\mathbf{x}$ and $\mathbf{y}$ axes.



(more questions on the next page)

- (3a) Suppose we have 3 black and 3 red pens, and we select 2 pens in random.
 Let $A = \{\textit{the first pen is red}\}$ and $B = \{\textit{the second pen is red}\}$.
 Find if these events are independent.

Please write your answer here:

$\mathbb{P}(A) =$	$\mathbb{P}(B) =$	$\mathbb{P}(A \cap B) =$	Are A and B independent?
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- 3(b) Find the probability the second pen is red, given that the first pen is red.

Please write your answer here:

$\mathbb{P}(B A) =$

(more questions on the next page)

- (3c) Suppose again that we have 3 black and 3 red pens, and we select 2 pens in random. Let $\mathbf{X}$ be the number of red pens. Find the probability mass function for $\mathbf{X}$.

Please write your answer here:

$$\mathbb{P}(\mathbf{X} = 0) =$$

$$\mathbb{P}(\mathbf{X} = 1) =$$

$$\mathbb{P}(\mathbf{X} = 2) =$$

- 3(d) Find $\mathbb{E}\mathbf{X}$ and $\mathbf{Var}(\mathbf{X})$.

Please write your answer here:

$$\mathbb{E}\mathbf{X} =$$

$$\mathbf{Var}(\mathbf{X}) =$$

(more questions on the next page)

Optional problem for extra credit. Suppose that currently **0.2** of population is infected with flu. We have a test with overall error rate α , so that α is the false positive rate, and also is the false negative rate. Assume that if we administer this test to a random person, and it is positive, then the probability that this person has the flu is **0.8**

What is α ?

Please write your answer here:

$\alpha =$

end of the test