MATH 3160 - Probability - Fall 2017

Quiz 10, Wednesday November 8

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are needed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. Use the notation $\boldsymbol{\Phi(x)}$ for the $\boldsymbol{\mathcal{N}(0,1)}$ distribution function, that is $\boldsymbol{\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)}$ where $\boldsymbol{Z}$ is the standard normal random variable.

(1) Let $\boldsymbol{X}$ be a uniform random variable on the interval $\boldsymbol{[0,4]}$, and $\boldsymbol{Y = \sqrt{X}}$. Find the c.d.f. $\boldsymbol{F_Y(y)}$ of $\boldsymbol{Y}$. *Hint: it may be useful if first you find the range of* $\boldsymbol{Y}$*, and use cases to define the c.d.f.*

(2) Find the p.d.f. $\boldsymbol{f_Y(y)}$ of $\boldsymbol{Y}$.

(3) Find $\mathbb{E}\boldsymbol{Y}$

(4) Find $\mathbb{E}\boldsymbol{Y^2}$

(... more questions on the other side of the paper ...)

(5) If $\boldsymbol{Z}$ is the standard normal $\boldsymbol{\mathcal{N}(0,1)}$ random variable, find the c.d.f. and the p.d.f. of $\boldsymbol{|Z|}$.
Hint: you can use function $\boldsymbol{\Phi(x)}$ *and cases.*

Extra credit question: what is $\mathbb{E}|\boldsymbol{Z}|$ and $\mathbf{Var}\,|\boldsymbol{Z}|$?

Please write ***Your name:*** _____