MATH 3160 - Probability - Fall 2017

Test 2, Wednesday November 15

Please write ***Your name:*** ____________________________

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are needed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. ***Two two-sided hand-written pages of notes are allowed.***

. . . .

(1) Two balls are withdrawn randomly without replacement from a bowl containing **3** white and **3** black balls. Let $\boldsymbol{X}$ be the number of white balls among the withdrawn balls. What are the probability mass function of $\boldsymbol{X}$, $\mathbb{E}\boldsymbol{X}$ and $\mathbf{Var}\,(\boldsymbol{X})$?

Please write your answer here:

p.m.f.:

$\mathbb{E}\boldsymbol{X} =$

$\mathbf{Var}\,(\boldsymbol{X}) =$

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(2) Suppose that earthquakes occur on the West coast of the U.S. on average at a rate of 3 per week (including very mild ones) and follow Poisson probability distribution. What is the probability that there will be 2 earthquakes next week, if we suppose that at least one will happen? *(Hint: use conditional probability).*

Please write your answer here:

$\boldsymbol{P(X = 2 \mid X \geqslant 1) =}$

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(3) Suppose $\boldsymbol{X}$ is exponentially distributed with the mean $\mathbb{E}\boldsymbol{X} = \mathbf{2}$. What is the probability $\mathbf{3} < \boldsymbol{X} < \mathbf{5}$ if we know that $\boldsymbol{X} > \mathbf{2}$? *(Hint: use conditional probability and the basic properties of the exponentially distribution).*

Please write your answer here:

$\boldsymbol{P(3 < X < 5 \| X > 2) =}$

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(4) Suppose $\boldsymbol{X = \mathcal{N}(\mu, \sigma^2)}$, $\boldsymbol{P(X < 0) = 0.15866 = \Phi(-1)}$ and $\boldsymbol{P(X < 5) = 0.97725 = \Phi(2)}$. Find $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$.

Please write your answer here:

$\boldsymbol{\mu =}$	$\boldsymbol{\sigma =}$

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(5) Suppose we toss a fair coin **16** times. Find the formula for the best possible normal approximation of the probability that there are at least **9** heads. You do not have to evaluate the numeral value but your answer should include $\boldsymbol{\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy} = \mathbb{P}(\boldsymbol{Z < x})$, where $\boldsymbol{Z}$ is the standard normal random variable.

Please write your answer here:

$\boldsymbol{P(X} \geqslant 9) \approx$

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(6) Suppose the random variable $\boldsymbol{X}$ is uniformly distributed in the interval $[\mathbf{0}, \mathbf{2}]$ and $\boldsymbol{Y} = \boldsymbol{X}^{\mathbf{3}}$. Find the c.d.f. $\boldsymbol{F_Y(y)}$ and $\mathbb{E}\boldsymbol{Y}$.

Please write your answer here:

$\boldsymbol{F_Y(y)} =$	$\mathbb{E}\boldsymbol{Y} =$