

Show all work.

Write each problem on a separate page. Each answer should be clearly written in the end of the page. Preferably, make a 7-page pdf file and submit in HuskyCT.

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(1a) Let \mathbf{X}, \mathbf{Y} be two independent geometric random variables with $\mathbf{p} = 1/2$. Find $\mathbb{P}(\mathbf{max}(\mathbf{X}, \mathbf{Y}) \leq 2)$ and $\mathbb{P}(\mathbf{min}(\mathbf{X}, \mathbf{Y}) \leq 2)$.

(1b) In the same situation find let $\mathbf{U} = \mathbf{max}(\mathbf{X}, \mathbf{Y})$ and $\mathbf{V} = \mathbf{min}(\mathbf{X}, \mathbf{Y})$. Find the joint probability mass function of \mathbf{U} and \mathbf{V} .

(2a) Let \mathbf{X}, \mathbf{Y} be two independent exponential random variables with $\lambda = 1$. Find $\mathbb{P}(\mathbf{X} + \mathbf{Y} < 1)$.

(2b) Find $\mathbb{P}(\mathbf{X} - \mathbf{Y} < 1)$.

(2c) Let $\mathbf{U} = \mathbf{X} + \mathbf{Y}$ and $\mathbf{V} = \mathbf{X} - \mathbf{Y}$. Find the joint probability density function of \mathbf{U} and \mathbf{V} .

(2d) In the same situation find the conditional probability density function of \mathbf{U} given \mathbf{V} : find $f_{\mathbf{U}|\mathbf{V}}(\mathbf{u}, \mathbf{v})$.

(2e) In the same situation find the conditional probability density function of \mathbf{V} given \mathbf{U} : find $f_{\mathbf{V}|\mathbf{U}}(\mathbf{v}, \mathbf{u})$.

end of the quiz