

Show all work.

Write each problem on a separate page. Each answer should be clearly written in the end of the page. Preferably, make a single pdf file and submit in HuskyCT.

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Required problem:

- (1a) Let \mathbf{X}, \mathbf{Y} be uniformly distributed in the triangle defined by $x < 2, y < 1, x + 2y > 2$. Find the marginal densities $f_{\mathbf{X}}(x)$ and $f_{\mathbf{Y}}(y)$.
- (1b) In the same situation find $\mathbb{E}(\mathbf{X}|\mathbf{Y})$ and $\mathbb{E}(\mathbf{Y}|\mathbf{X})$.

Extra credit problems:

- (1c) In the same situation find $\rho(\mathbf{X}, \mathbf{Y})$.
- (2a) Let $\mathbf{U} = e^{\mathbf{X}+\mathbf{Y}}$ and $\mathbf{V} = e^{\mathbf{X}}$ where \mathbf{X}, \mathbf{Y} are two independent exponential random variables with parameter $\lambda > 0$. Find $\text{Cov}(\mathbf{U}, \mathbf{V})$ if $\lambda = 3$.
- (2b) In the same situation find how $\text{Cov}(\mathbf{U}, \mathbf{V})$ depends on λ . Is there λ for which $\text{Cov}(\mathbf{U}, \mathbf{V}) = 0$?

End of the quiz