Show all work. A correct answer with no solution will give only a partial credit.
Write each problem on a separate page. Each answer should be clearly written in the end of the page.
Preferably, make a single pdf file and submit in HuskyCT.

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(1) Find the moment generation function of $m_X(t) = \mathbb{E}e^{tX}$ for a random variable X with the probability density $f(x) = (x-1)/2$ when $1 < x < 3$ and zero otherwise. Also, find $m_X(0)$, $m_X'(0)$, $m_X''(0)$ by computing the moments of X.

(2) Look at problems $2(a,b,c)$ in Test 2 given last week: find the moment generating function $m_X(t) = \mathbb{E}e^{tX}$ and the joint moment generating function $m_{X,Y}(s,t) = \mathbb{E}e^{tX+sY}$.
Extended Hint: if you use the change of variables $u = x+y,\ v = x-y$, you can solve this problem using the table of distributions, without computing any integrals.

$$\mathbb{E}e^{tX+sY} = \mathbb{E}e^{t(U+V)/2+s(U-V)/2} = \mathbb{E}e^{\alpha U+\beta V}$$

To finish this solution you need to do the following:

- compute α, β.
- justify why $\mathbb{E}e^{\alpha U+\beta V} = \mathbb{E}e^{\alpha U}\mathbb{E}e^{\beta V}$ using the solution for Test 2
- find formulas for $\mathbb{E}e^{\alpha U}$ and $\mathbb{E}e^{\beta V}$ using the solution for Test 2, and the table of distributions
- use this to obtain the formula for $m_{X,Y}(s,t)$
- use this to obtain the formula for $m_X(t)$

Extra credit question: in the situation of problems $2(a,b,c)$ in Test 2 given last week, use moment generating functions to confirm that $\mathbb{E}X = 5/4$ and $\mathbb{E}XY = 5/3$, and to compute $\mathbb{E}X^2$ and $\mathbb{E}X^3$.
Hint: it may be helpful to use Taylor expansions.

End of the quiz