Show all work. A correct answer with no solution will give only a partial credit.
Write each problem on a separate page. Each answer should be clearly written in the end of the page.
Preferably, make a single pdf file and submit in HuskyCT.

(1) Find the moment generation function of $m_X(t) = \mathbb{E}e^{tX}$ for a random variable X with the probability density $f(x) = (x-1)/2$ when $1 < x < 3$ and zero otherwise. Also, find $m_X(0)$, $m'_X(0)$, $m''_X(0)$ by computing the moments of X.

(2) Look at problems $\mathbf{2(a,b,c)}$ in Test 2 given last week: find the moment generating function $m_X(t) = \mathbb{E}e^{tX}$ and the joint moment generating function $m_{X,Y}(s,t) = \mathbb{E}e^{tX+sY}$.

Hint: if you use the change of variables $u = x + y$, $v = x - y$, you can solve this problem using the table of distributions, without computing any integrals.

$x = (u+v)/2 \quad y = (u-v)/2$

Extra credit question: * in the situation of problems $\mathbf{2(a,b,c)}$ in Test 2 given last week, use moment generating functions to confirm that $\mathbb{E}X = 5/4$ and $\mathbb{E}XY = 5/3$, and to compute $\mathbb{E}X^2$ and $\mathbb{E}X^3$.

Hint: it may be helpful to use Taylor expansions.

End of the quiz

Ex: X is unif., $f = 1/2$

$$m(t) = \int_1^3 \frac{1}{2} e^{tx}\,dx = \frac{1}{2t}\left(e^{3t} - e^{t}\right) \leftarrow t=0$$

$$\mathbb{E}X = \int_1^3 x\cdot\frac{1}{2}\cdot dx = \frac{1}{4}(9-1) = 2 = m'(0) \;?$$

Taylor

$$m(t) = \frac{1}{2t}\left(1 + 3t + \frac{9t^2}{2} + \dots - 1 - t - \frac{t^2}{2} - \dots\right) = 1 + \frac{8t^2}{4t} + \dots$$

$$\underbrace{m(t) = m(0) + t\,m'(0) + \dots}_{*} \qquad \boxed{m(0) = 1} \qquad \underbrace{= 1 + 2t + \dots}_{*}$$

$$\boxed{m'(0) = 2}$$

$(fg)' = f'g + fg'$ $\Rightarrow$ $\int f'g = fg - \int fg'$

$$\int \underset{g}{x}\,\underset{f'}{e^x}\,dx = xe^x - \int e^x = xe^x - e^x + C = e^x(x-1)$$