

Show all work. A correct answer with no solution will give only a partial credit.

Write each problem on a separate page. Each answer should be clearly written in the end of the page.

Preferably, make a single pdf file and submit in HuskyCT.

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Let $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \dots$ be independent Poisson random variables with parameter $\lambda = 2$.

Let $\mathbf{S}_n = \mathbf{X}_1 + \mathbf{X}_2 + \dots + \mathbf{X}_n$

(1a) Write a formula for the probability $\mathbb{P}(\mathbf{S}_{50} = 101)$.

(1b) Then use a calculator to compute this probability with accuracy up to five decimal places.

(2a) Write a formula for the normal approximation to $\mathbb{P}(\mathbf{S}_{50} = 101)$ using the continuity correction.

(2b) Compute compute this approximation using the normal table, with accuracy up to five decimal places.

Hint: your answer in (2b) should be close to, but not exactly the same, as the answer in (1b).

Extra credit question: Prove that *if X_n are Geometric $\text{Geom}(p_n = \frac{1}{n})$ then*

$$\frac{1}{n\lambda} X_n \rightarrow \text{Exp}(\lambda) \text{ as } n \rightarrow \infty \text{ in (probability) distribution.}$$

Use geometric series to show $\mathbb{P}(\frac{1}{n\lambda} X_n > x) \rightarrow e^{-\lambda x} = P(X > x)$ if $X = \text{Exp}(\lambda)$, and also show that the moment generating functions converge.

End of the quiz