**Show all work**. A correct answer with no solution will give only a partial credit.
Write each problem on a separate page. Each answer should be clearly written in the end of the page.
Preferably, make a single pdf file and submit in HuskyCT.

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Let $X_1, X_2, X_3, ...$ be independent Poisson random variables with parameter $\lambda = 2$.
Let $S_n = X_1 + X_2 + ... + X_n$

(1a) Write a formula for the probability $\mathbb{P}(S_{50} = 101)$.
(1b) Then use a calculator to compute this probability with accuracy up to five decimal places.
$Answer: \; S_{50} \sim Pois(100) \;\; therefore \;\; \mathbb{P}(S_{50} = 101) = 100^{101}/(e^{100}(101!)) = 0.03947$

(2a) Write a formula for the normal approximation to $\mathbb{P}(S_{50} = 101)$ using the continuity correction.
(2b) Compute this approximation using the normal table, with accuracy up to five decimal places.
Hint: your answer in (2b) should be close to, but not exactly the same, as the answer in (1b).
$Answer: \; \mathbb{P}(100.5 < 100 + 10Z < 101.5) = \Phi(0.15) - \Phi(0.05) \approx 0.55966 - 0.51994 = 0.03972$

***Extra credit question***: Prove that $\;if\; X_n \; are \; Geometric \; Geom(p_n = \frac{1}{n}) \; then$

$$\frac{1}{n\lambda} X_n \to Exp(\lambda) \; as \; n \to \infty \; in \; \text{(probability)} \; distribtion.$$

Use geometric series to show $\mathbb{P}(\frac{1}{n\lambda} X_n > x) \to e^{-\lambda x} = P(X > x)$ if $X = Exp(\lambda)$, and also show that the moment generating functions converge.

$Solution\ 1:$ First, assume $\lambda = 1$. Then

$$\mathbb{P}(\frac{1}{n\lambda} X_n > x) = \mathbb{P}(X_n > nx) = p_n \sum_{k=[nx]}^{\infty} (1 - p_n)^k = p_n \frac{(1 - p_n)^{[xn]}}{1 - (1 - p_n)} = \left(1 - \frac{1}{n}\right)^{[xn]} \xrightarrow[n \to \infty]{} e^{-x}$$

$Solution\ 2:$ To compute the moment generating functions for $\frac{1}{n} X_n$, we can use the Taylor series as follows

$$m_n(t) = \frac{p_n e^{t/n}}{1 - (1 - p_n) e^{t/n}} = \frac{1}{ne^{-t/n} - n + 1} = \frac{1}{n(1 - t/n + t^2/n^2 + ...) - n + 1} =$$
$$= \frac{1}{n - t + t^2/n - n + 1 + ...} \xrightarrow[n \to \infty]{} \frac{1}{1 - t}$$

With another $\lambda$ the computations are very similar.

***End of the quiz***