

(1) Find $\text{Var}(X)$ if $P(X=1)=1/2$ and $P(X=2)=P(X=4)=1/4$

$$\mathbb{E}X = 2, \mathbb{E}X^2 = 5.5, \text{Var } X = 1.5$$

(2) Find $\text{Var}(Y)$ if $P(Y=2)=1/2$ and $P(Y=4)=P(Y=8)=1/4$

$$\mathbb{E}X = 4, \mathbb{E}X^2 = 22, \text{Var } X = 6$$

(3) Suppose that, for a virus test, the true positive rate is 0.6 and the false positive rate is 0.01. If we know that the rate of infection in the population is 0.04, then what is the rate of positive tests in this population?

$$0.6 \cdot 0.04 + 0.01 \cdot (1 - 0.04) = 0.0336$$

Challenge question for extra credit: if, in a different population, we do not know the rate of infection, but we know that the rate of positive tests is 0.04, then what is the rate of infection that gives this rate of positive tests?

$$\text{solving } 0.6x + 0.01(1 - x) = 0.04 \text{ we obtain } x = \frac{3}{59} \approx 0.05085$$

End of the quiz