In a fictional country, the congress consists of 400 members of the house of representatives, and 100 senators. There are two political parties, A and B. The house of representatives and the senate are both equally split between these two parties. They decided to take all decisions at random.

(1) In how many ways can they choose the president and vice present of the senate, if there are no restrictions on their party affiliation?

Solution: $100 \cdot 99$

(2) In how many ways can they choose the president and vice present of the senate, if they agreed to be from different parties?

Solution: $100 \cdot 50 = 2 \cdot 50^2$

(3) In the house of representatives, how many ways are to select a committee of 20 members, if it has to be equally split between the parties A and B?

Solution: $\binom{200}{10}^2$

(4) In the house of representatives, 2 members are chosen at random. What is the probability that they will be from different parties?

Solution: $\frac{200}{399} = \left( \frac{200}{1} \right)^2 / \binom{400}{2} = \frac{2 \cdot 200^2}{400 \cdot 399}$

(5) Suppose 3 senate members are chosen at random. What is the probability that they will be from one party?

Solution: $\frac{49 \cdot 48}{99 \cdot 98} = \binom{2}{1} \cdot \binom{50}{3} / \binom{100}{3}$