Solutions

You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. NO calculators are needed.

(1) Suppose that we are to assign 12 police officers to 5 patrols, 4 in station, 3 in schools. How many different assignments can we organize?

Answer:

$$\frac{12!}{5! \cdot 4! \cdot 3!} = \binom{12}{5} \cdot \binom{7}{4}$$

This is similar to Examples 1.28 and 1.29 in the textbook. Note that the answer is correct if the officers are assigned in groups. If the officers are assigned individually to different locations, then the number of assignments is 12!.

(2) Suppose that we are to assign 12 police officers to 4 patrols, 3 in station, 2 in schools, and remaining to traffic control. How many different assignments can we organize in this case?

Answer:

$$\frac{12!}{4! \cdot 3! \cdot 2! \cdot 3!} = \binom{12}{4} \cdot \binom{8}{3} \cdot \binom{5}{2}$$

Again, this is similar to Examples 1.28 and 1.29 in the textbook. Note that the answer is correct if the officers are assigned in groups and if the officers are assigned individually to different locations, then the number of assignments is 12!.

(3) Expand $(2ab + 3)^4$ using the binomial theorem.

Answer:

$$\sum_{k=0}^{4} \binom{4}{k} (2ab)^k 3^{4-k} = \binom{4}{0} 3^4 + \binom{4}{1} (2ab)3^3 + \binom{4}{2} (2ab)^2 3^2 + \binom{4}{3} (2ab)^3 3 + \binom{4}{4} (2ab)^4$$

$$= 3^4 + 4 \cdot (2ab)3^3 + 6 \cdot (2ab)^2 3^2 + 4 \cdot (2ab)^3 3 + (2ab)^4$$

(4) A password can be made up of 2 digits and 2 letters.

(a) How many different passwords are possible?

Answer: $\binom{4}{2} \cdot 10^2 \cdot 26^2 = 6 \cdot 10^2 \cdot 26^2$

(b) How many are possible if all the digits are odd?

Answer: $\binom{4}{2} \cdot 5^2 \cdot 26^2 = 6 \cdot 5^2 \cdot 26^2$

(c) How many can be made in which digits are different and letters are different?

Answer: $\binom{4}{2} \cdot 10 \cdot 9 \cdot 26 \cdot 25 = 6 \cdot 10 \cdot 9 \cdot 26 \cdot 25 = \binom{10}{2} \cdot \binom{26}{2} \cdot 4!$

Note that $\binom{4}{2} = 6$ is the number of choices for positions for digits.