Show all steps.

Let $\boldsymbol{X}$ be exponentially distributed and $\mathbb{E}\boldsymbol{X} = \mathbf{1/2}$

(1) What is $\mathbb{P}(\,\mathbf{2<X<5 \mid X>1}\,)$?

here $\boldsymbol{\lambda = 2}$

$\mathbb{P}(\,\mathbf{2<X<5 \mid X>1}\,) = \mathbb{P}(\,\mathbf{2-1<X<5-1}\,) =$

$\mathbb{P}(\,\mathbf{1<X<4}\,) = \boldsymbol{e^{-2} - e^{-8}}$

another correct solution: $\mathbb{P}(\,\mathbf{2<X<5 \mid X>1}\,) = \frac{e^{-4}-e^{-10}}{e^{-2}}$

(2) What is $\mathbb{E}\boldsymbol{X^2}$?

$\mathbb{E}\boldsymbol{X^2} = (\mathbb{E}\boldsymbol{X})^2 + \mathbf{Var}\,\boldsymbol{X} = \mathbf{1/4 + 1/4 = 1/2}$

another correct solution: $\mathbb{E}\boldsymbol{X^2} = \int_0^\infty \mathbf{2}\boldsymbol{x^2 e^{-2x} dx} =$

$\boldsymbol{x^2 e^{-2x}}\Big|_0^\infty + \int_0^\infty \mathbf{2}\boldsymbol{x e^{-2x} dx} = \mathbf{0} + \boldsymbol{x e^{-2x}}\Big|_0^\infty + \int_0^\infty \boldsymbol{e^{-2x} dx} =$

$\mathbf{0 + 0} - \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{e^{-2x}}\Big|_0^\infty = \frac{\mathbf{1}}{\mathbf{2}}$

(3) What is the probability density function of $\boldsymbol{Y = \sqrt{X}}$?

$\boldsymbol{P(Y > y) = P(\sqrt{X} > y) = P(X > x) = e^{-2x} = e^{-2y^2}}$

by the chain rule $\boldsymbol{f_Y(y) = 4ye^{-2y^2}}$

another correct solution uses Theorem 11.1 from the textbook with $\boldsymbol{y = g(x) = \sqrt{x}}$ and $\boldsymbol{x = y^2}$:

$\boldsymbol{f_Y(y) = \frac{f_X(x)}{|g'(x)|} = \frac{2e^{-2x}}{\frac{1}{2\sqrt{x}}} = 4\sqrt{x}e^{-2x} = 4ye^{-2y^2}}$

End of the quiz