

Show all steps. You do not have to simplify

(1) If $f(x) = a/x^3$ for $x > 2$ and zero otherwise,

what is a ? What is the cumulative distribution function $F(x)$?

Solution: $\int_2^{\infty} 1/x^3 dx = -1/2x^2|_{x=2}^{\infty} = 1/8$, therefore $a = 8$

$$F(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 - 4/x^2 & \text{if } x \geq 2 \end{cases}$$

(2) If $f(x) = 2x/9$ for $0 < x < 3$ and zero otherwise,

what are $\mathbb{E}X$, $\mathbb{E}X^2$, $\text{Var}(X)$?

$$\mathbb{E}X = \int_0^3 2x^2/9 dx = 2$$

$$\mathbb{E}X^2 = \int_0^3 2x^3/9 dx = 9/2$$

$$\text{Var}(X) = 9/2 - 2^2 = 1/2$$

Optional questions for extra credit: find $\int_0^{\infty} xe^{-x} dx$ and $\int_0^{\infty} xe^{-x^2} dx$

$$\int xe^{-x} dx = -(1+x)e^{-x} + c \quad \int xe^{-x^2} dx = -e^{-x^2}/2 + c$$

$$\int_0^{\infty} xe^{-x} dx = 1 \quad \int_0^{\infty} xe^{-x^2} dx = 1/2$$

End of the quiz