Show all steps.

https://alexander-teplyaev.uconn.edu/2020/11/05/white-board-2020-11-02/

Let $\boldsymbol{X, Y}$ be independent ***exponentially distributed*** with $\boldsymbol{\lambda = 1}$

(1) What is $\mathbb{P}(\boldsymbol{Y < 2X})$?

Answer : $\mathbb{P}(\boldsymbol{Y < 2X}) = \boldsymbol{2/3}$ see white-board solution pictures

(2) If $\boldsymbol{U = X + Y}$ and $\boldsymbol{V = X - Y}$,
what is the joint probability density function $\boldsymbol{f_{U,V}(u, v)}$?

Answer : $\boldsymbol{f_{U,V}(u, v) = \frac{1}{2}e^{-u}}$ when $\boldsymbol{-u < v < u}$ and $\boldsymbol{0}$ otherwise, see white-board solution pictures

Optional question for extra credit: if $\boldsymbol{X, Y}$ are independent ***standard normal*** random variables, find $\mathbb{P}(\boldsymbol{|X| < Y})$

Answer : $\mathbb{P}(\boldsymbol{|X| < Y}) = \boldsymbol{1/4}$ see white-board solution pictures

End of the quiz