Show all steps.

see https://alexander-teplyaev.uconn.edu/2020/11/10/white-board-2020-11-09/

Let $\boldsymbol{X, Y}$ be ***uniformly distributed*** on the triangle $\mathbf{0 < x < y < 3}$

(1) What is $\mathbb{E}\boldsymbol{X}$?

$$\mathbb{E}\boldsymbol{X} = \frac{\mathbf{2}}{\mathbf{9}} \int_{\mathbf{0}}^{\mathbf{3}} \int_{\mathbf{0}}^{\boldsymbol{y}} \boldsymbol{x}\, \boldsymbol{dxdy} = \mathbf{1}$$

(2) What is $\mathbb{E}\boldsymbol{Y}$?

$$\mathbb{E}\boldsymbol{Y} = \frac{\mathbf{2}}{\mathbf{9}} \int_{\mathbf{0}}^{\mathbf{3}} \int_{\mathbf{0}}^{\boldsymbol{y}} \boldsymbol{y}\, \boldsymbol{dxdy} = \mathbf{2}$$

(3) What is $\mathbb{E}\boldsymbol{XY}$?

$$\mathbb{E}\boldsymbol{XY} = \frac{\mathbf{2}}{\mathbf{9}} \int_{\mathbf{0}}^{\mathbf{3}} \int_{\mathbf{0}}^{\boldsymbol{y}} \boldsymbol{xy}\, \boldsymbol{dxdy} = \frac{\mathbf{9}}{\mathbf{4}}$$

(4) What is $\mathbf{Cov}(\boldsymbol{X}, \boldsymbol{Y})$?

$$\frac{\mathbf{9}}{\mathbf{4}} - \mathbf{2} \cdot \mathbf{1} = \frac{\mathbf{1}}{\mathbf{4}}$$

Optional question for extra credit: what are $\mathbb{E}(\boldsymbol{X}|\boldsymbol{Y})$ and $\mathbb{E}(\boldsymbol{Y}|\boldsymbol{X})$?

$$\boldsymbol{f_Y}(\boldsymbol{y}) = \frac{\mathbf{2}\boldsymbol{y}}{\mathbf{9}} \qquad \boldsymbol{f_X}(\boldsymbol{x}) = \frac{\mathbf{6} - \mathbf{2}\boldsymbol{x}}{\mathbf{9}}$$

$$\mathbb{E}(\boldsymbol{X}|\boldsymbol{Y}) = \frac{\boldsymbol{y}}{\mathbf{2}} \qquad \mathbb{E}(\boldsymbol{Y}|\boldsymbol{X}) = \frac{\mathbf{3}}{\mathbf{2}} + \frac{\boldsymbol{x}}{\mathbf{2}}$$

End of the quiz