Show all steps.

see

https://alexander-teplyaev.uconn.edu/2020/11/30/white-board-2020-11-16/

Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be two independent random variables with respective moment generating functions

$$\boldsymbol{m_X(t) = \frac{1}{1-t}, \quad m_Y(t) = \frac{1}{(1-t)^2}, \quad \text{if } t < 1.}$$

(1) What is the moment generating function $\boldsymbol{m_{X+Y}(t)}$ of $\boldsymbol{X+Y}$?

$$\boldsymbol{m_{X+Y}(t) = \frac{1}{(1-t)^3}, \quad \text{if } t < 1.}$$

(2) What is $\mathbb{E}\,(\boldsymbol{X+Y})$?

$$\boldsymbol{\mathbb{E}\,(X+Y) = 3}$$

(3) What is $\mathbb{E}\,(\boldsymbol{X+Y})^{\boldsymbol{2}}$?

$$\boldsymbol{\mathbb{E}\,(X+Y) = 12}$$

(4) What is $\mathbf{Var}\,(\boldsymbol{X+Y})$?

$$\boldsymbol{\mathbf{Var}\ (X+Y) = 3}$$

End of the quiz