

Show all work. A correct answer with no solution will give only a partial credit.

Write each problem on a separate page. Each answer should be clearly written in the end of the page. Preferably, make a single pdf file and submit in HuskyCT.

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Let $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \dots$ be independent Poisson random variables with parameter $\lambda = \frac{1}{2}$.
Let $\mathbf{S}_n = \mathbf{X}_1 + \mathbf{X}_2 + \dots + \mathbf{X}_n$

- (1a) Write a formula for the probability $\mathbb{P}(|\mathbf{S}_{50} - 25| \leq 1)$.
(1b) Then use a calculator to compute this probability with accuracy up to five decimal places.

We have that \mathbf{S}_{50} is Poisson with $\lambda = 25$.

$$\mathbb{P}(|\mathbf{S}_{50} - 25| \leq 1) = \mathbb{P}(\mathbf{S}_{50} = 24) + \mathbb{P}(\mathbf{S}_{50} = 25) + \mathbb{P}(\mathbf{S}_{50} = 26) =$$

$$\frac{25^{24}e^{-25}}{24!} + \frac{25^{25}e^{-25}}{25!} + \frac{25^{26}e^{-25}}{26!} = 0.23551$$

- (2a) Write a formula for the normal approximation to $\mathbb{P}(|\mathbf{S}_{50} - 25| \leq 1)$ using the continuity correction.
(2b) Compute this approximation using the normal table, with accuracy up to five decimal places.
Hint: your answer in (2b) should be close to, but not exactly the same, as the answer in (1b).

We have $n = 50, n\mu = 25, n\sigma^2 = 25$

$$\mathbb{P}(|\mathbf{S}_{50} - 25| \leq 1) = \mathbb{P}(23.5 < \mathbf{S}_{50} < 26.5) \approx$$

$$\mathbb{P}(23.5 < 25 + 5Z < 26.5) = \mathbb{P}(-0.3 < Z < 0.3) = 2\Phi(0.3) - 1 = 0.23582$$

End of the quiz